

University Of Alberta



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Teachers' Edition

# Investigating School Mathematics

Extending  
the Ideas

QA

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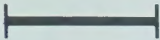

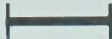

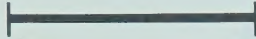



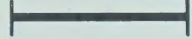



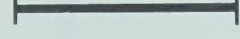

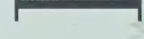

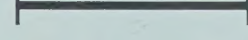

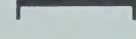

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Complete the table.

	IF this is one unit	THEN the length of this object is	
1.			5 units
2.			6 units
3.			3 units
4.			6 units
5.			4 units
6.			7 units
7.			3 units
8.			6 units
9.			3 units
10.			6 units

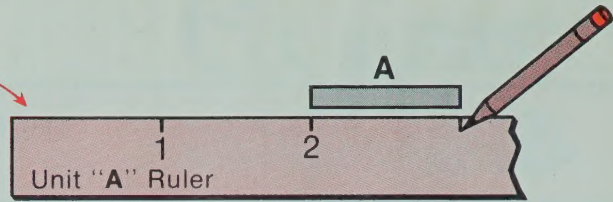
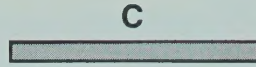
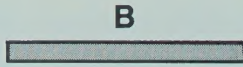
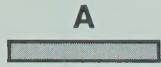
Give children freedom and challenge to figure out how to do this measuring. Possibilities are, marking the unit along the edge of a slip of paper, or using a compass to see how many of the units are contained in the object.



## ● Making a Ruler

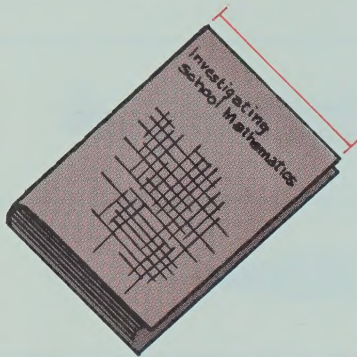
Make a ruler.

Use one of the units—A, B, or C, shown below.



Use your ruler to measure some of the objects suggested in the pictures.

1.



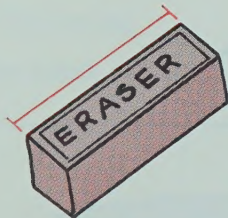
answers will vary \_\_\_\_\_ units

2.



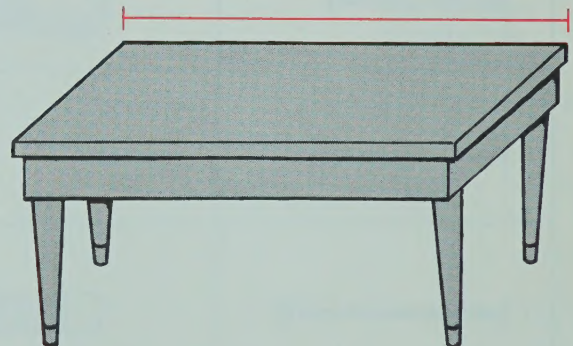
answers will vary \_\_\_\_\_ units

3.



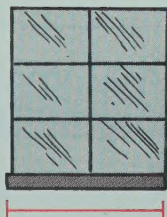
answers will vary \_\_\_\_\_ units

4.



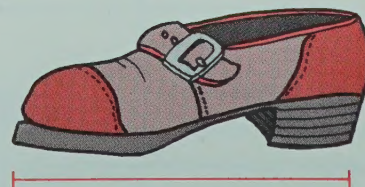
answers will vary \_\_\_\_\_ units

5.



answers will vary \_\_\_\_\_ units

6.



answers will vary \_\_\_\_\_ units

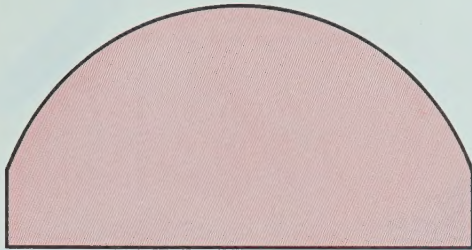
Provide help as needed in constructing the ruler. Children can check their results by comparing them with someone else who chose the same unit. Be sure that children are measuring objects suggested by the pictures rather than measuring the pictures on the page.



## ● Finding the Distance Around

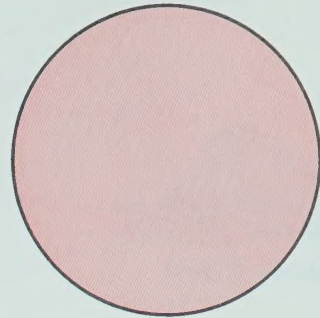
1. Can you find a way to measure the distance around these figures? Use centimeter units.

A



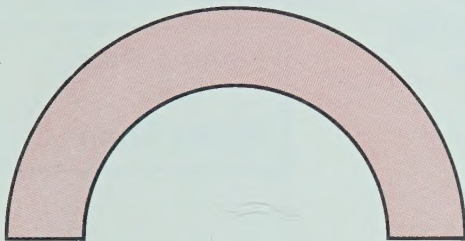
About 16 cm

B



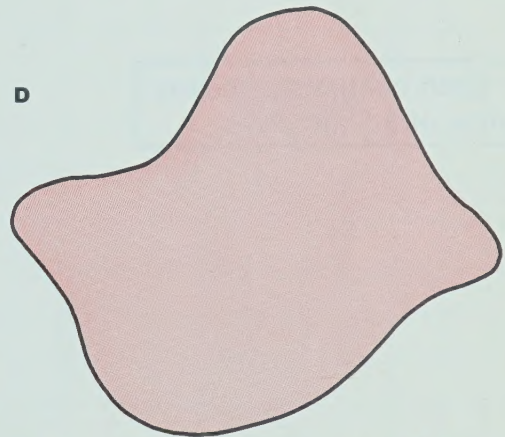
About 12½ cm

C



About 18 cm

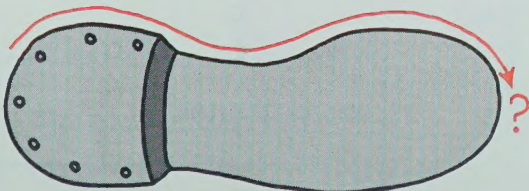
D



About 19 cm

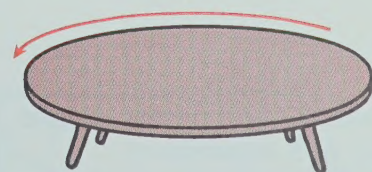
2. Can you find the distance around some objects such as the ones suggested by these pictures?

A



A Shoe: about \_\_\_\_\_ cm *answers will vary*

B

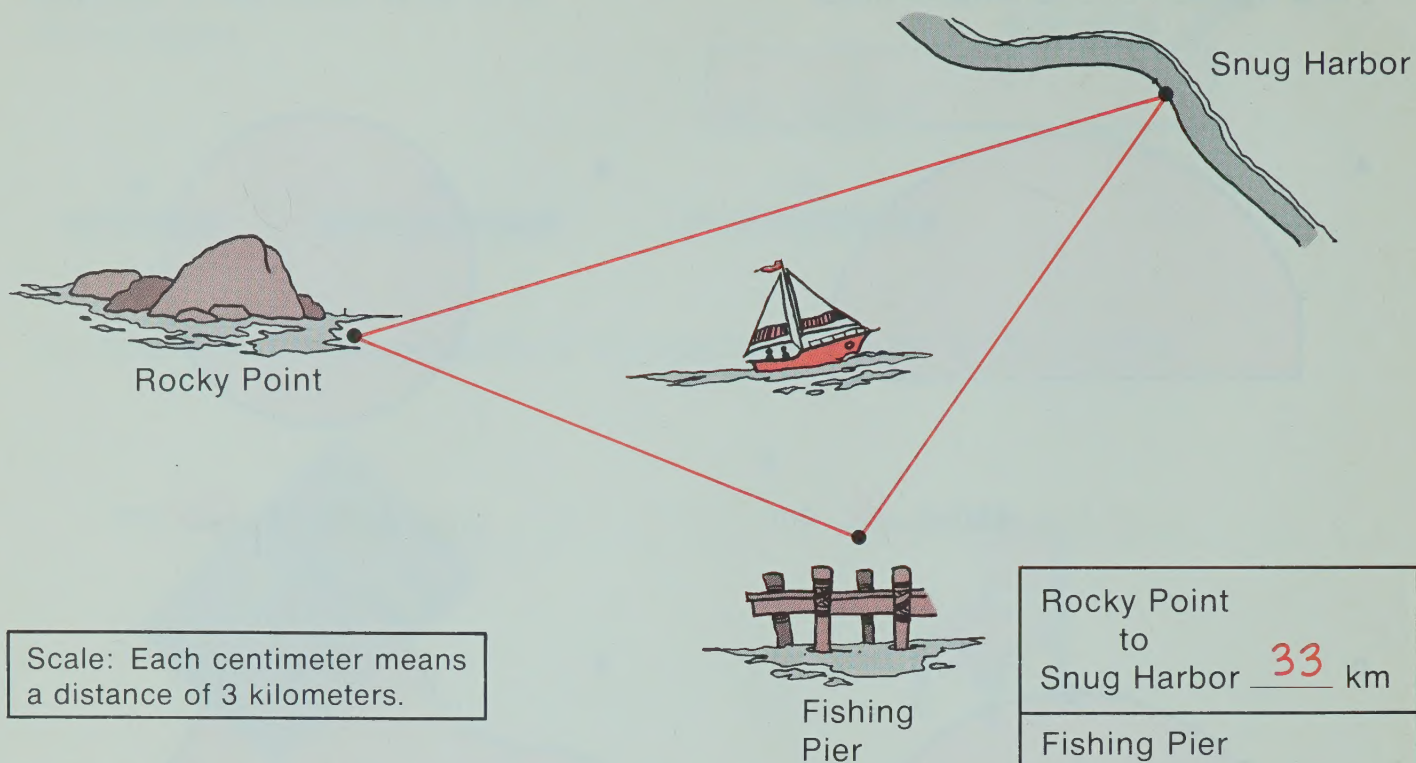


A Round Object: about \_\_\_\_\_ cm *answers will vary*

Give children freedom to figure out how to measure these distances. Some possibilities are, string, tape measure, and cutouts.

## ● Distance on a Map

Use the map and scale to give the distances in kilometers.



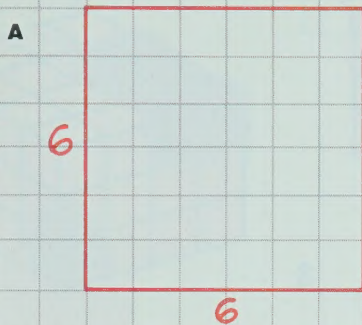
Have children attempt these exercises involving scale drawings with a minimum of explanation. Just be sure they understand how to read the scale and then challenge them to find the distances.



## ●Area

1. Can you draw regions that have the given areas?  
Use the graph paper unit.

Square

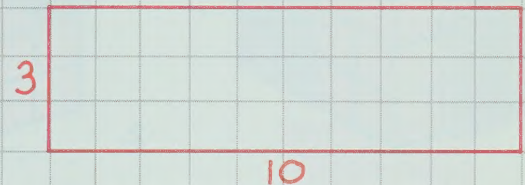


Area: 36 square units

Rectangle

B

answers may vary

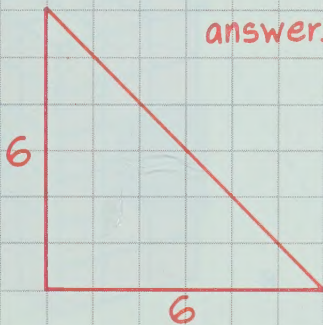


Area: 30 square units

Triangle

C

answers may vary

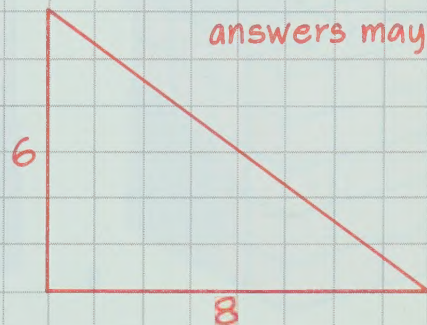


Area: 18 square units

Triangle

D

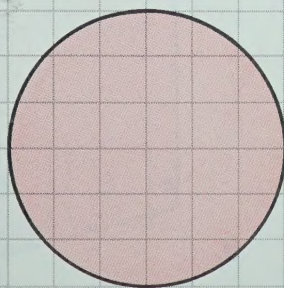
answers may vary



Area: 24 square units

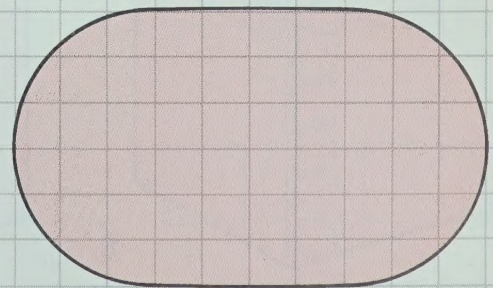
2. Estimate the area of each region.

A



28 square units

B



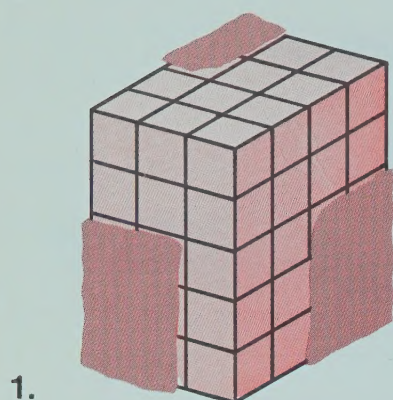
52 square units

If children make mistakes in drawing the figures in exercise 1, have them find the area of the figure they drew before attempting a second trial. In exercise 2 estimates within three or four units of the given number are satisfactory.

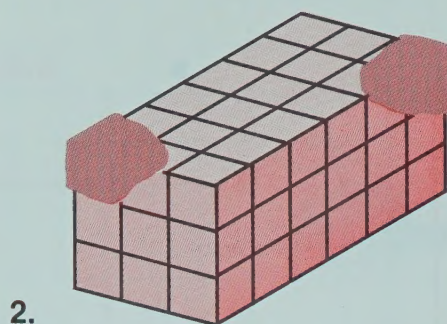


# ● Volume

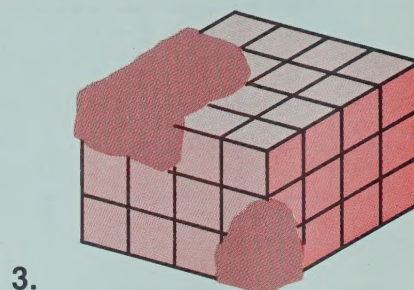
Find the volume of each figure.



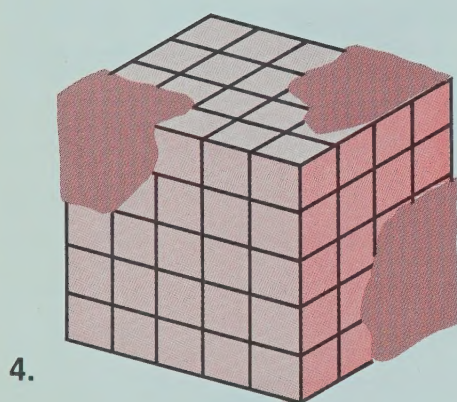
60 cubic units



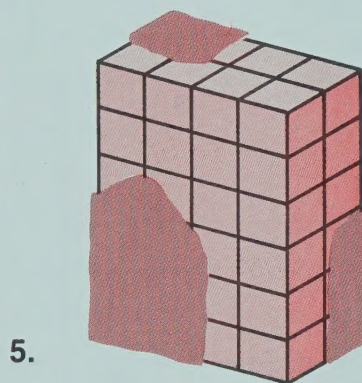
54 cubic units



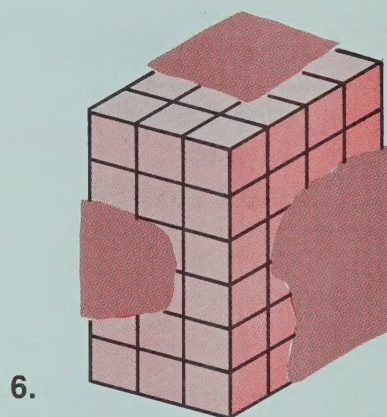
48 cubic units



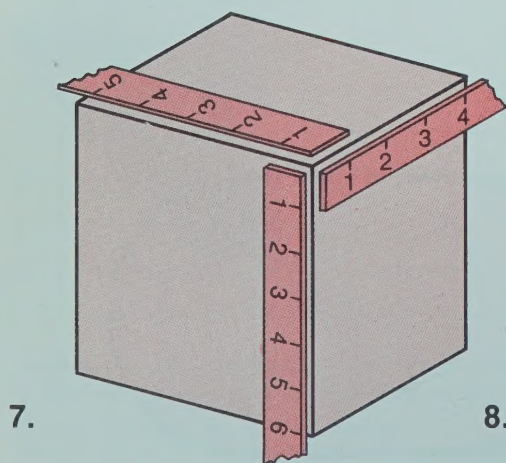
100 cubic units



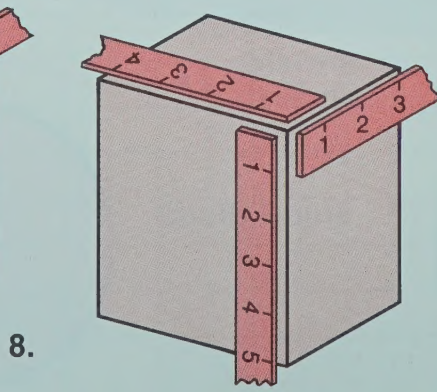
48 cubic units



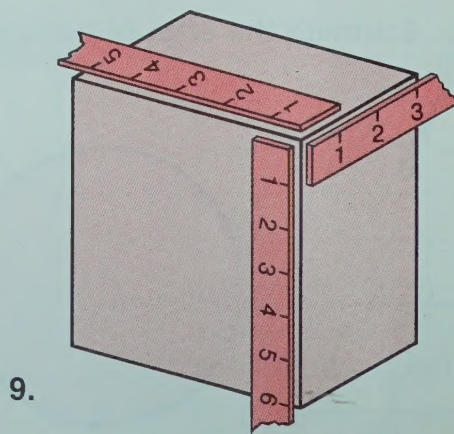
72 cubic units



120 cubic units



60 cubic units



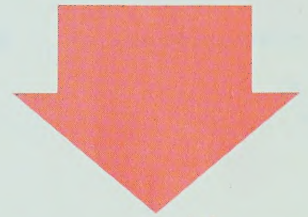
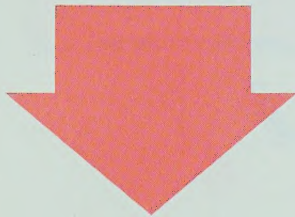
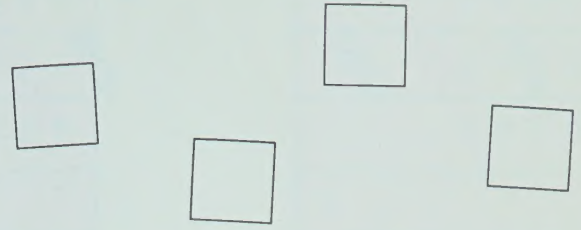
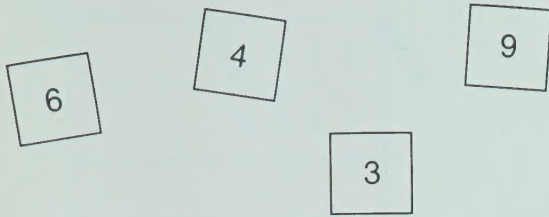
90 cubic units

The figures in exercise 1 through 6 are partially covered to encourage children to find the volume without actually counting. 7, 8, and 9 are attempts to find whether or not the child has made a discovery for finding volume.



The numbers below were built with the digits 6, 3, 4, 9. Complete the matching.

Choose your own set of digits. Use your digits to build numbers to match with the colored boxes.



answers will vary

9 3 6 4

9 6 4 3

3 4 6 9

6 4 9 3

4 9 6 3

Largest

Fewest  
number of  
hundreds

Closest to  
5000

Largest  
number of  
tens

Smallest

•

•

•

•

•

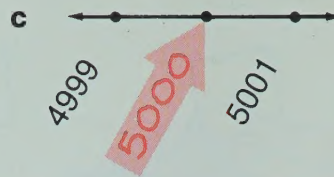
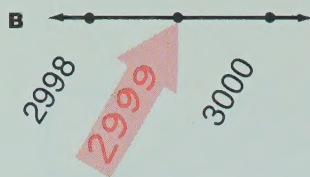
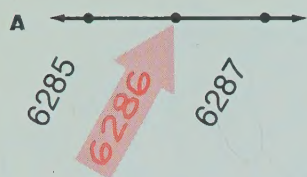


## ● Order of Numbers

Give the missing numbers.

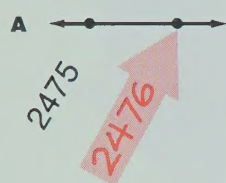
1.

Between



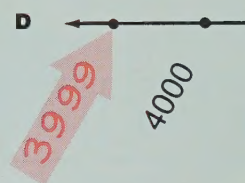
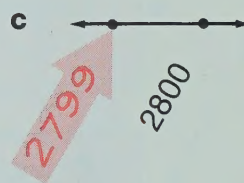
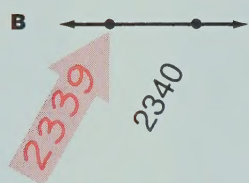
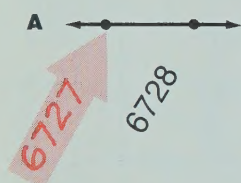
2.

After



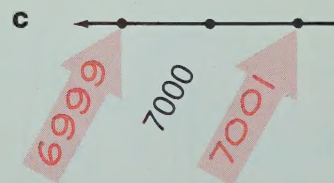
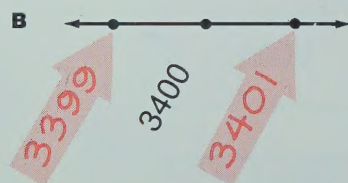
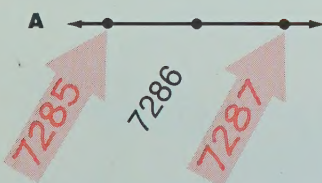
3.

Before



4.

Before and After



5.

Make some of your own

answers will vary



Be sure the children understand that they are finding the whole numbers just before and just after the given numbers.



## Sequences with Large Numbers

Give the next three numbers in each table.

1.

3 2 6 5
3 2 6 6
3 2 6 7
<b>A</b> 3268
<b>B</b> 3269
<b>C</b> 3270

2.

4 5 9 6
4 5 9 7
4 5 9 8
<b>A</b> 4599
<b>B</b> 4600
<b>C</b> 4601

3.

2 9 9 7
2 9 9 8
2 9 9 9
<b>A</b> 3000
<b>B</b> 3001
<b>C</b> 3002

4.

1 6,4 5 2
1 6,4 5 4
1 6,4 5 6
<b>A</b> 16,458
<b>B</b> 16,460
<b>C</b> 16,462

5.

3 5,6 2 7
3 5,7 2 7
3 5,8 2 7
<b>A</b> 35,927
<b>B</b> 36,027
<b>C</b> 36,127

6.

2 5,3 8 4
3 0,3 8 4
3 5,3 8 4
<b>A</b> 40,384
<b>B</b> 45,384
<b>C</b> 50,384

7.

2 6 8,4 8 3
2 7 8,4 8 3
2 8 8,4 8 3
<b>A</b> 298,483
<b>B</b> 308,483
<b>C</b> 318,483

8.

7 4 8,4 8 9
7 4 8,6 8 9
7 4 8,8 8 9
<b>A</b> 749,089
<b>B</b> 749,289
<b>C</b> 749,489

9.

6 0 3,1 2 3
6 5 3,1 2 3
7 0 3,1 2 3
<b>A</b> 753,123
<b>B</b> 803,123
<b>C</b> 853,123

Help the children understand that they are looking for special patterns in finding the numbers in these sequences. For example, in exercise 5 the digit in the hundreds place increases.



## ● Large Numbers

1. With the numerals on these cards, you can “write” six different large numbers. How many of them can you write?

302

475

961

A 302, 475, 961

B 302, 961, 475

C 475, 302, 961

D 475, 961, 302

E 961, 302, 475

F 961, 475, 302

2. A Which of the six numbers above is largest? 961, 475, 302

B Which is smallest? 302, 475, 961

3. Give the correct sign  $>$  or  $<$  for each .

A 32,645,794  33,000,000


K 1,000,000,000  99,999,999

B 500,000,000  498,634,259

L 212,341,003  78,867,948


C 6,137,000  5,879,000

M 367,482,531  367,482,529

D 364,287,512  400,000,000

N 675,835,609  675,834,609

E 98,463,217  642,123,012

O 407,826,742  407,816,742

4. Make up six 9-digit numbers of your own. *Answers will vary.*

A \_\_\_\_\_

B \_\_\_\_\_

C \_\_\_\_\_

D \_\_\_\_\_

E \_\_\_\_\_

F \_\_\_\_\_

5. Arrange your numbers above in order from smallest to largest. *Answers will vary.*

A \_\_\_\_\_  
smallest

B \_\_\_\_\_

C \_\_\_\_\_

D \_\_\_\_\_

E \_\_\_\_\_

F \_\_\_\_\_  
largest

Actually making slips of paper that contain the digits will help children understand the ideas involved in these exercises.



## Which Number is Larger?

Part of each slip of paper is missing. Each slip had two 8-digit numbers on it. When you can decide which of the two numbers is greater, ring it. When you can't decide, ring both numbers.

1.

3 8,2 8
3 4,3 0

2.

4,9 8 7
1 7,6 2 5

3.

6 8,	,7 8 3
6 8,4	,1 3 2

4.

,2 8 7,0 0 0
,3 0 0,0 0 0

5.

2 6,4 8 9,3 4 7
2 6,4 9 0,3 4

6.

7 5,4 8 9,6 5
7 5,4 8 9,6 5 3

7.

3 9,	,4 2 7
4 1,	,2 0 4

8.

7 6
6 9,8 4 7,

9.

8
8 2,3 4 7,6 4

10.

7 8 4,6 4 2
3 1,7 8 4,6 4 2

11.

6 8,2 3	4 4
6 8,2 3	4 3

12.

7 5,2 8 7,6 4 2
3,2 8 7,6 4 2

13.

9 9,9 9 9,9 9 9	
9 9,	9 8

14.

1 0,	1 0
1 0,0 0 0,0 0 0	

15.

8,6 4 3,2 8 7
9 2,6 4 3,2 8 7

16.

9 6,2 3 0,0 0 0
5,2 3 0,0 0 0

17.

3 6,4 0 7,
3 6,4 1 7,3 2

18.

9 8,3	2 7
9 8,4 6	,2 7 0

As children work on these exercises you may need to remind them that they are to think of each number as an 8-digit number. It will sometimes be instructive to have the children give sample numbers for a particular exercise, as if the digits weren't missing.



## ● Using Powers of 10 in Place Value

1. Study the chart. Then give the correct sign  $>$  or  $<$  for each

$10^2 = 100$	$10^6 = 1,000,000$
$10^3 = 1000$	$10^7 = 10,000,000$
$10^4 = 10,000$	$10^8 = 100,000,000$
$10^5 = 100,000$	

A  $98,765 < 10^5$

B  $98,765 < 10^6$

C  $10^8 > 34,178,654$

D  $10^7 < 34,178,654$

E  $10^6 < 9,999,999$

F  $10^6 > 999,999$

G  $10^8 < 111,111,111$

H  $100,000,001 > 10^8$

I  $999,999,999 > 10^8$

J  $99,999,999 < 10^8$

K  $10^7 < 88,888,888$

L  $10^6 < 8,888,888$

2. A What number is one more than  $10^5$ ? 100,001

B What number is 675 more than  $10^3$ ? 1675

C What number is one less than  $10^7$ ? 9,999,999

D What number is 7463 more than  $10^6$ ? 1,007,463

E What number is one less than  $10^8$ ? 99,999,999

3. A Write the number for  $10^9$ . 1,000,000,000

B Write the number that is one less than  $10^9$ . 999,999,999

4. Find the sums.

EXAMPLE:  $10^4 + 10^2 = 10,100$

A  $10^3 + 10^2 = 1100$

F  $10^8 + 10^6 + 10^4 + 10^2 = 101,010,100$

B  $10^4 + 10^3 = 11,000$

G  $10^2 - 1 = 99$

C  $10^5 + 10^3 = 101,000$

H  $10^3 - 1 = 999$

D  $10^4 + 10^3 + 10^2 = 11,100$

I  $10^6 - 1 = 999,999$

E  $10^5 + 10^4 + 10^3 + 10^2 = 111,100$

Be sure children understand the notation introduced at the top of the page. A helpful hint as to the use of this notation is that the exponent indicates the number of zeros.



Give the missing numbers or function rule.

1. Function Rule

Double and Add 4

Input	Output
5	14
3	10
7	18
A 6	16
B 8	20
C 9	22

2. Function Rule

Double and Subt 3

Input	Output
2	1
7	11
6	9
A 5	7
B 4	5
C 9	15

3. Function Rule

A Subtract 3

Input	Output
7	4
13	10
6	3
3	0
B 8	5
C 5	2

4. Function Rule

A Double

Input	Output
6	12
8	16
7	14
B 9	18
C 5	10
D 4	8

5. Function Rule

A Double and Add 1

Input	Output
6	13
5	11
7	15
B 9	19
C 3	7
D 4	9

6. Function Rule

A Double and Subtract 4

Input	Output
2	0
4	4
7	10
B 6	8
C 8	12
D 9	14

7. Function Rule

A Multiply by 3

Input	Output
6	18
2	6
5	15
B 4	12
C 3	9
D 0	0








8. Function Rule

0 if even, 1 if odd








Input	Output
7	1
4	0
5	1
8	0
9	1
2	0

Give the missing number pair for each row.

9. Sum  Difference

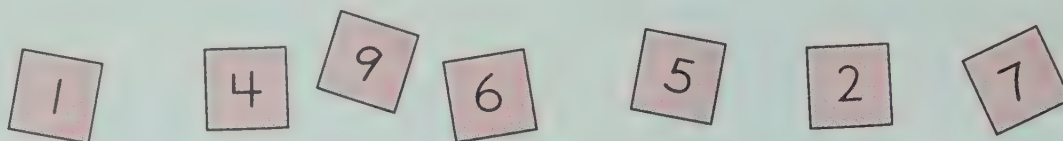
	11	6		5	1
	12	9		3	6
A	13	7		6	1
B	17	9		8	1
C	15	8		7	1
D	14	8		6	2
E	13	8		5	3

10. Sum  Difference

A	12	6		6	0
B	18	9		9	0
C	16	8		8	0
D	14	7		7	0
E	10	9		1	8
F	9	8		1	7
G	8	7		1	6



## ● Adding and Checking



Can you find 3 numbers on the cards whose sum is this number?

Answers may vary.



Your first equation is correct if the sum of the numbers on the other cards is this number. Check to see.

1. A  $\underline{4} + \underline{5} + \underline{6} = 15$

B  $\underline{1} + \underline{9} + \underline{2} + \underline{7} = 19$

Try it again with these sums.

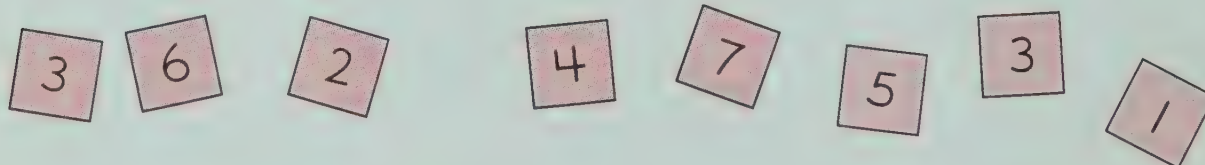
2. A  $\underline{9} + \underline{7} + \underline{1} = 17$

B  $\underline{4} + \underline{6} + \underline{5} + \underline{2} = 17$

3. A  $\underline{9} + \underline{5} + \underline{2} = 16$

B  $\underline{1} + \underline{4} + \underline{6} + \underline{7} = 18$

Try it with a new set of cards.



First Equation: answers may vary

Check:

4. A  $\underline{6} + \underline{4} + \underline{7} = 17$

B  $\underline{5} + \underline{3} + \underline{3} + \underline{2} + \underline{1} = 14$

5. A  $\underline{7} + \underline{4} + \underline{1} = 12$

B  $\underline{3} + \underline{6} + \underline{2} + \underline{5} + \underline{3} = 19$

6. A  $\underline{7} + \underline{6} + \underline{3} = 16$

B  $\underline{2} + \underline{4} + \underline{5} + \underline{3} + \underline{1} = 15$

7. A  $\underline{5} + \underline{6} + \underline{2} = 13$

B  $\underline{3} + \underline{4} + \underline{7} + \underline{3} + \underline{1} = 18$

Make a set of cards and equation pairs of your own. Try them on a classmate.

Each time a child finds a sum on the left he should follow immediately by checking on the right.



## Finding Sums

1. Find the sums.

$$\begin{array}{r} \text{A} \quad 37 \\ + 85 \\ \hline 122 \end{array}$$

$$\begin{array}{r} \text{B} \quad 68 \\ + 92 \\ \hline 160 \end{array}$$

$$\begin{array}{r} \text{C} \quad 764 \\ + 849 \\ \hline 1613 \end{array}$$

$$\begin{array}{r} \text{D} \quad 967 \\ + 868 \\ \hline 1835 \end{array}$$

$$\begin{array}{r} \text{E} \quad 7643 \\ + 4829 \\ \hline 12,472 \end{array}$$

$$\begin{array}{r} \text{F} \quad 763 \\ \quad 842 \\ + 976 \\ \hline 2581 \end{array}$$

$$\begin{array}{r} \text{G} \quad 934 \\ \quad 821 \\ + 109 \\ \hline 1864 \end{array}$$

$$\begin{array}{r} \text{H} \quad 846 \\ \quad 738 \\ + 426 \\ \hline 2010 \end{array}$$

$$\begin{array}{r} \text{I} \quad 7427 \\ \quad 8368 \\ + 7210 \\ \hline 23,005 \end{array}$$

$$\begin{array}{r} \text{J} \quad 6243 \\ \quad 9605 \\ + 7876 \\ \hline 23,724 \end{array}$$

$$\begin{array}{r} \text{K} \quad 6374 \\ \quad 8675 \\ + 3294 \\ \hline 18,343 \end{array}$$

$$\begin{array}{r} \text{L} \quad 8467 \\ \quad 5396 \\ + 9810 \\ \hline 23,673 \end{array}$$

$$\begin{array}{r} \text{M} \quad 7654 \\ \quad 8339 \\ \quad 7465 \\ + 8376 \\ \hline 31,834 \end{array}$$

$$\begin{array}{r} \text{N} \quad 9462 \\ \quad 8487 \\ \quad 6538 \\ + 9567 \\ \hline 34,054 \end{array}$$

$$\begin{array}{r} \text{O} \quad 35,678 \\ \quad 97,605 \\ \quad 38,765 \\ + 92,768 \\ \hline 264,816 \end{array}$$

2. Make some addition problems of your own that have these sums.  
Have a classmate check your problems.

$$\begin{array}{r} \text{A} \\ + \\ \hline 134 \end{array}$$

$$\begin{array}{r} \text{B} \\ + \\ \hline 1463 \end{array}$$

$$\begin{array}{r} \text{C} \\ + \\ \hline 1962 \end{array}$$

$$\begin{array}{r} \text{D} \\ + \\ \hline 15,030 \end{array}$$

$$\begin{array}{r} \text{E} \\ + \\ \hline 192 \end{array}$$

$$\begin{array}{r} \text{F} \\ + \\ \hline 2364 \end{array}$$

$$\begin{array}{r} \text{G} \\ + \\ \hline 2865 \end{array}$$

$$\begin{array}{r} \text{H} \\ + \\ \hline 29,030 \end{array}$$

Exercise 1 can be used as readiness for exercise 2. If children are particularly adept with addition, they can go immediately to exercise 2 without solving exercise 1.



## ● Magic Squares

1. Complete each square to make it a magic square.

**A**

8	9	4
3	7	11
10	5	6

**B**

12	5	10
7	9	11
8	13	6

**C**

10	9	14
15	11	7
8	13	12

**D**

14	9	13	2
4	11	7	16
3	12	8	15
17	6	10	5

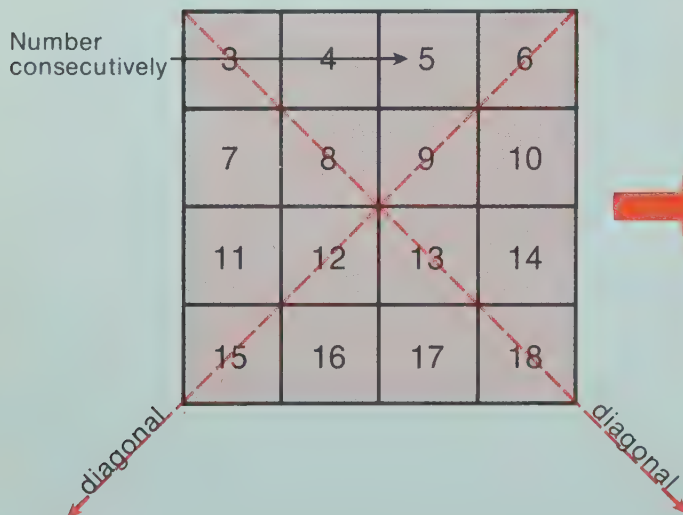
**E**

8	13	9	20
18	11	15	6
19	10	14	7
5	16	12	17

**F**

15	1	2	12
4	10	9	7
8	6	5	11
3	13	14	0

2. Follow directions to make your own 4 by 4 magic square.



18	4	5	15
7	13	12	10
11	9	8	14
6	16	17	3

Start with a blank square

- A** Put in the numbers not on the colored lines (shown in black above).
- B** Reverse the diagonals as shown in red above.

*answers will vary*

3. Now you make a magic square using a different set of consecutive numbers.

Make it clear to the child that he needs to start with two four-by-four squares. Then number one of them consecutively and put the numbers as directed into the second square. Note that once the non-diagonal numbers are put into the blank squares, the diagonals can be put in simply by reversing the numbers; upper left to lower right with the other two squares along that diagonal also exchanged.

## Subtracting

1. Find the differences.

$$\begin{array}{r} \text{A} \quad 72 \\ - 37 \\ \hline 35 \end{array}$$

$$\begin{array}{r} \text{B} \quad 346 \\ - 179 \\ \hline 167 \end{array}$$

$$\begin{array}{r} \text{C} \quad 6302 \\ - 1445 \\ \hline 4857 \end{array}$$

$$\begin{array}{r} \text{D} \quad 7435 \\ - 896 \\ \hline 6539 \end{array}$$

$$\begin{array}{r} \text{E} \quad 30,265 \\ - 17,283 \\ \hline 12,982 \end{array}$$

$$\begin{array}{r} \text{F} \quad 1000 \\ - 6 \\ \hline 994 \end{array}$$

$$\begin{array}{r} \text{G} \quad 10,000 \\ - 1 \\ \hline 9999 \end{array}$$

$$\begin{array}{r} \text{H} \quad 100,000 \\ - 100 \\ \hline 99,900 \end{array}$$

$$\begin{array}{r} \text{I} \quad 100,000 \\ - 900 \\ \hline 99,100 \end{array}$$

$$\begin{array}{r} \text{J} \quad 100,000 \\ - 10,101 \\ \hline 89,899 \end{array}$$

$$\begin{array}{r} \text{K} \quad 604 \\ - 138 \\ \hline 466 \end{array}$$

$$\begin{array}{r} \text{L} \quad 700 \\ - 248 \\ \hline 452 \end{array}$$

$$\begin{array}{r} \text{M} \quad 6003 \\ - 5276 \\ \hline 727 \end{array}$$

$$\begin{array}{r} \text{N} \quad 70,304 \\ - 13,678 \\ \hline 56,626 \end{array}$$

$$\begin{array}{r} \text{O} \quad 60,003 \\ - 7,438 \\ \hline 52,565 \end{array}$$

$$\begin{array}{r} \text{P} \quad 50,901 \\ - 12,493 \\ \hline 38,408 \end{array}$$

2. Solve the equations.

$$\begin{array}{l} \text{A} \quad 3465 - 1783 = n \\ n = 1682 \end{array}$$

$$\begin{array}{l} \text{B} \quad 6204 - 3466 = n \\ n = 2738 \end{array}$$

$$\begin{array}{l} \text{C} \quad 7004 - 838 = n \\ n = 6166 \end{array}$$

$$\begin{array}{l} \text{D} \quad 9682 - 99 = n \\ n = 9583 \end{array}$$

$$\begin{array}{l} \text{E} \quad 4003 - 764 = n \\ n = 3239 \end{array}$$

$$\begin{array}{l} \text{F} \quad 7765 - 693 = n \\ n = 7072 \end{array}$$

3. Make some subtraction problems of your own that have these differences. Have a classmate check your problems. *answers will vary*

$$\begin{array}{r} \text{A} \quad \underline{\hspace{1cm}} \\ 344 \end{array}$$

$$\begin{array}{r} \text{B} \quad \underline{\hspace{1cm}} \\ 28 \end{array}$$

$$\begin{array}{r} \text{C} \quad \underline{\hspace{1cm}} \\ 73 \end{array}$$

$$\begin{array}{r} \text{D} \quad \underline{\hspace{1cm}} \\ 1642 \end{array}$$

$$\begin{array}{r} \text{E} \quad \underline{\hspace{1cm}} \\ 307 \end{array}$$

$$\begin{array}{r} \text{F} \quad \underline{\hspace{1cm}} \\ 2834 \end{array}$$

$$\begin{array}{r} \text{G} \quad \underline{\hspace{1cm}} \\ 3076 \end{array}$$

$$\begin{array}{r} \text{H} \quad \underline{\hspace{1cm}} \\ 4369 \end{array}$$

Use exercise 1 and 2 only for those children who need extra practice prior to the challenging problems in exercise 3.



## ● Reconstruction Problems

1. Fill in the missing digits.

$$\begin{array}{r} \text{A} \quad 7 \text{ } 2 \text{ } 3 \\ - 1 \text{ } 5 \text{ } 8 \\ \hline 5 \text{ } 6 \text{ } 5 \end{array}$$

$$\begin{array}{r} \text{B} \quad 4 \text{ } 1 \text{ } 6 \\ - 3 \text{ } 4 \text{ } 2 \\ \hline 7 \text{ } 4 \end{array}$$

$$\begin{array}{r} \text{C} \quad 7 \text{ } 8 \text{ } 2 \text{ } 3 \\ - 1 \text{ } 5 \text{ } 6 \text{ } 0 \\ \hline 6 \text{ } 2 \text{ } 6 \text{ } 3 \end{array}$$

$$\begin{array}{r} \text{D} \quad 6 \text{ } 2 \text{ } 5 \text{ } 2 \\ - 1 \text{ } 3 \text{ } 4 \text{ } 9 \\ \hline 4 \text{ } 9 \text{ } 0 \text{ } 3 \end{array}$$

$$\begin{array}{r} \text{E} \quad 3 \text{ } 4 \text{ } 2 \\ - 1 \text{ } 0 \text{ } 6 \\ \hline 2 \text{ } 3 \text{ } 6 \end{array}$$

$$\begin{array}{r} \text{F} \quad 7 \text{ } 0 \text{ } 5 \\ - \quad 3 \text{ } 8 \\ \hline 6 \text{ } 6 \text{ } 7 \end{array}$$

$$\begin{array}{r} \text{G} \quad 5 \text{ } 6 \text{ } 3 \text{ } 8 \\ - \quad 1 \text{ } 5 \text{ } 9 \\ \hline 5 \text{ } 4 \text{ } 7 \text{ } 9 \end{array}$$


$$\begin{array}{r} \text{H} \quad 3 \text{ } 6 \text{ } 0 \text{ } 4 \\ - 1 \text{ } 6 \text{ } 6 \text{ } 4 \\ \hline 1 \text{ } 9 \text{ } 4 \text{ } 0 \end{array}$$

$$\begin{array}{r} \text{I} \quad 7 \text{ } 0 \text{ } 0 \\ - 1 \text{ } 0 \text{ } 5 \\ \hline 5 \text{ } 9 \text{ } 5 \end{array}$$

$$\begin{array}{r} \text{J} \quad 7 \text{ } 4 \text{ } 3 \\ - \quad 5 \text{ } 9 \\ \hline 6 \text{ } 8 \text{ } 4 \end{array}$$

$$\begin{array}{r} \text{K} \quad 7 \text{ } 0 \text{ } 6 \text{ } 0 \\ - \quad 4 \text{ } 8 \text{ } 8 \\ \hline 6 \text{ } 5 \text{ } 7 \text{ } 2 \end{array}$$

$$\begin{array}{r} \text{L} \quad 9 \text{ } 0 \text{ } 0 \text{ } 1 \\ - \quad 8 \text{ } 3 \text{ } 3 \\ \hline 8 \text{ } 1 \text{ } 6 \text{ } 8 \end{array}$$

2. Can you decide whether the problem is addition or subtraction, without seeing the missing digits? Put the correct sign (+) or (−) in the .

$$\begin{array}{r} \text{A} \quad \square \square \square \\ \text{+} \quad \square \square \square \\ \hline \square \square \square \end{array}$$

$$\begin{array}{r} \text{B} \quad \square \square \square \square \\ \text{+} \quad \square \square \square \\ \hline \square \square \square \end{array}$$

$$\begin{array}{r} \text{C} \quad \square \square \square \square \\ \text{+} \quad \square \square \square \square \\ \hline \square \square \square \end{array}$$

$$\begin{array}{r} \text{D} \quad \square \square \square \square \square \\ \text{+} \quad \square \square \square \square \\ \hline \square \square \square \square \square \end{array}$$

$$\begin{array}{r} \text{E} \quad \square \square \square \square \\ \text{+} \quad \square \square \square \square \\ \hline 9 \quad \square \square \square \end{array}$$

$$\begin{array}{r} \text{F} \quad \square \square \square \square \\ \text{+} \quad \square \square \square \square \\ \hline 1 \quad \square \square \square \end{array}$$

$$\begin{array}{r} \text{G} \quad \square \quad 6 \quad \square \\ \text{+} \quad \square \quad 0 \quad 8 \\ \hline \square \quad 5 \quad 3 \end{array}$$

$$\begin{array}{r} \text{H} \quad 9 \quad \square \square \square \\ \text{+} \quad 7 \quad \square \square \square \\ \hline \square \square \square \end{array}$$

Make it clear to the children that in exercise 2 they need not fill in missing digits. They need only to decide whether or not the problem is an addition or subtraction problem according to the digits or boxes they can see.

## Solving Problems

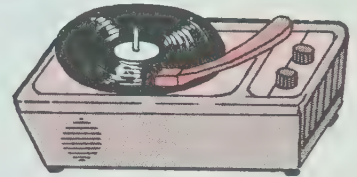
Read the problem carefully. Then put in numbers that make sense.  
Solve your problem. *answers will vary*

1. Jefferson School has \_\_\_\_\_ members in the band. Lincoln School has \_\_\_\_\_. How many more does Lincoln School have?

2. Ted had \_\_\_\_\_ tickets to sell for the school play.  
Lynn had \_\_\_\_\_. How many did they have in all?



3. Ginny had \_\_\_\_\_ records. She bought \_\_\_\_\_ new ones and gave away \_\_\_\_\_. How many does she have now?



4. Joe had \_\_\_\_\_ baseball cards. He gave \_\_\_\_\_ to Fred and \_\_\_\_\_ to Jerry. How many does he have left?



5. Mary had \_\_\_\_\_ dollars. She spent \_\_\_\_\_ for socks and \_\_\_\_\_ for gloves. How much does she have left?



6. Harry earned \_\_\_\_\_ for mowing a lawn and \_\_\_\_\_ cleaning the garage. He spent \_\_\_\_\_ for a snack. How much does he have left from his earnings?

7. Ann earned \_\_\_\_\_ in one week for baby sitting. She earned \_\_\_\_\_ the next week and \_\_\_\_\_ the third week. How much did she earn in three weeks?

8. Larry had \_\_\_\_\_ dollars. How much change should he get if he bought 2 games at \_\_\_\_\_ each?



Allow considerable freedom in terms of the numbers chosen. However, when numbers are unrealistic you should help the child make better selections.



## ● Writing Your Own Problems

Write and solve your own story problem for each picture. *answers will vary*

1.




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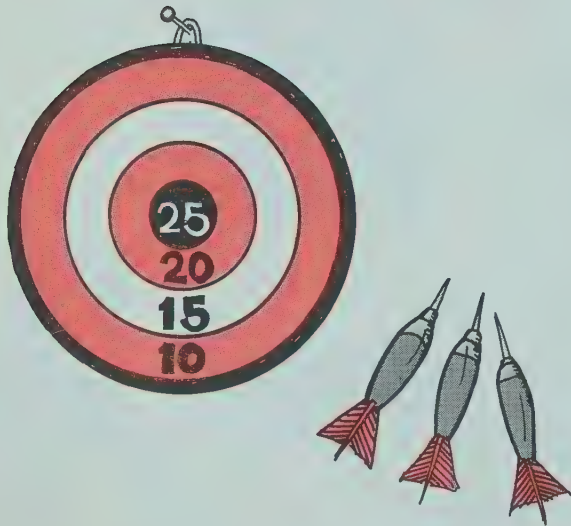
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2.




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3.

Population Figures		
City		Pop.
Marysville		23,487
Greensbury		56,103
Middletown		7,286
Rockton		103,785

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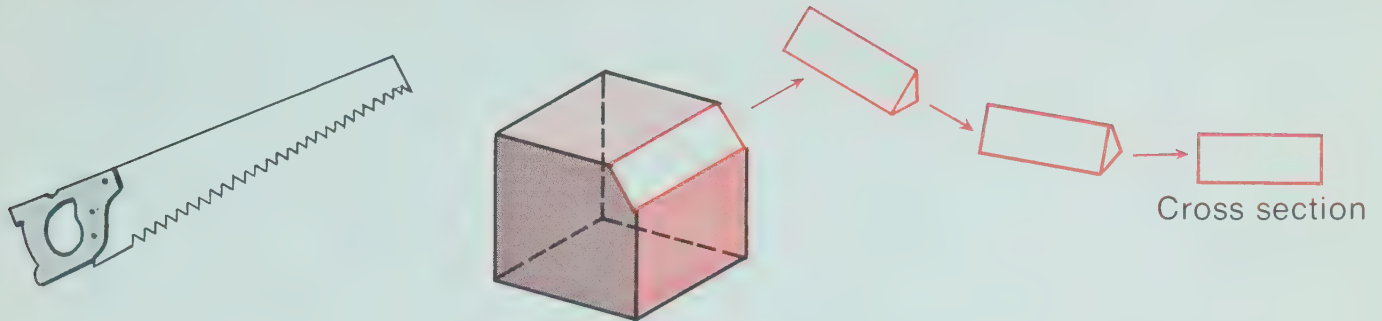
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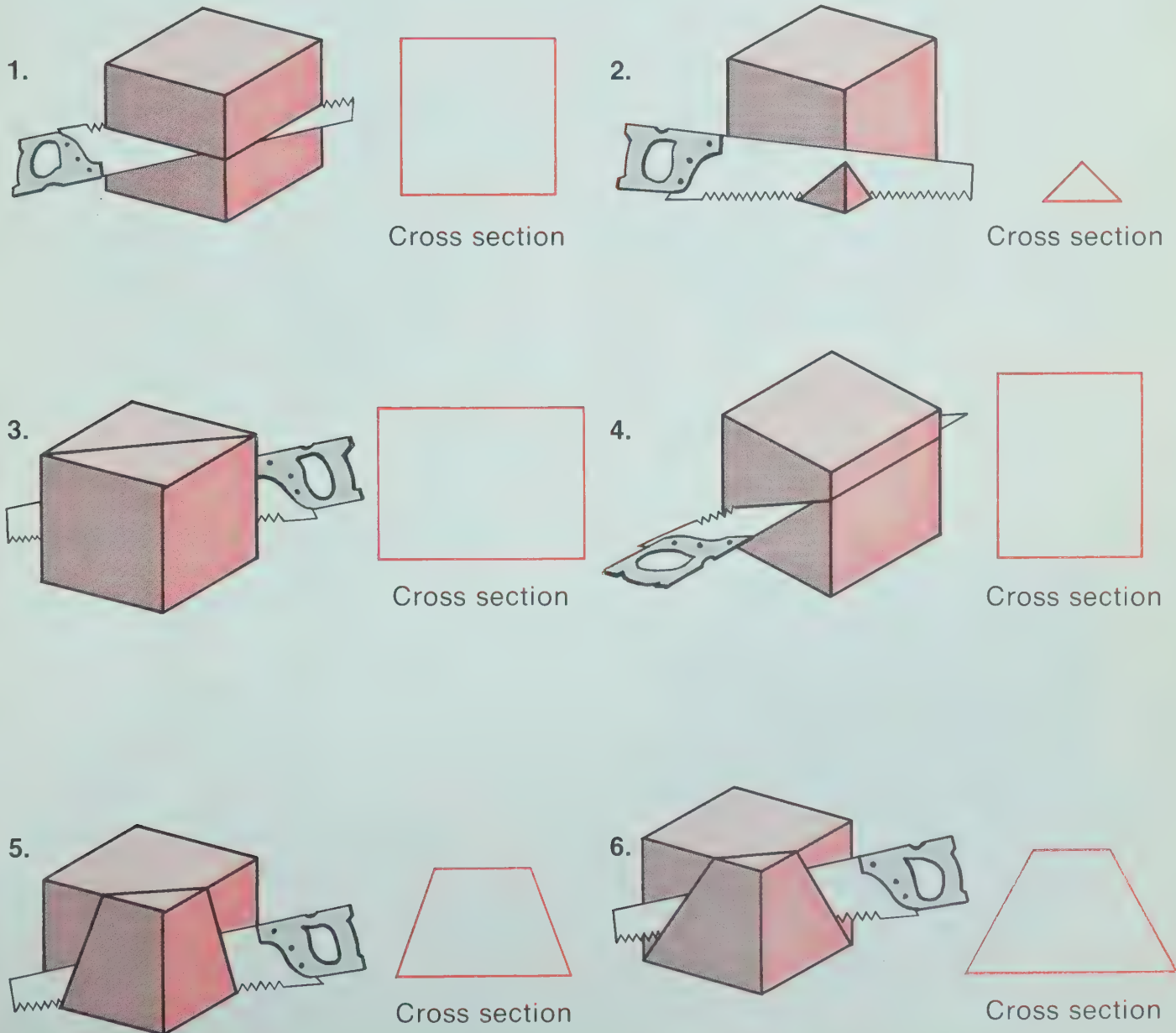
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Allow as much freedom as possible in terms of problems the children choose to write. Do not insist on any particular degree of difficulty.

The figure below explains a cross section.  
Think of cutting a piece off of a cube.



Draw a picture of the cross section for each cut shown below.  
Part 1 is an example.



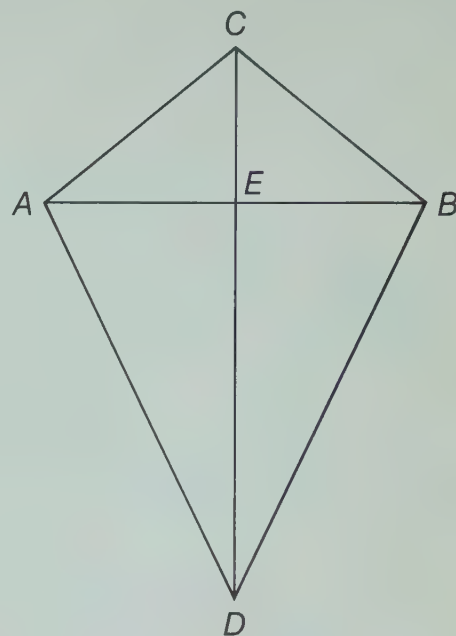
The ideal method for helping children understand this type of exercise, is to have them actually do the cutting. This can best be accomplished using a cube made out of clay or other material that can be easily cut according to the picture.



## ● Segments, Rays, and Angles

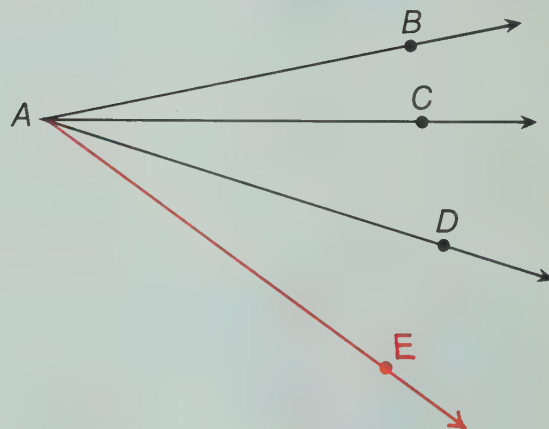
1. How many segments can you find and name on this figure?

AC    AE    AB    AD  
BC    BE    BD    CE  
CD    DE



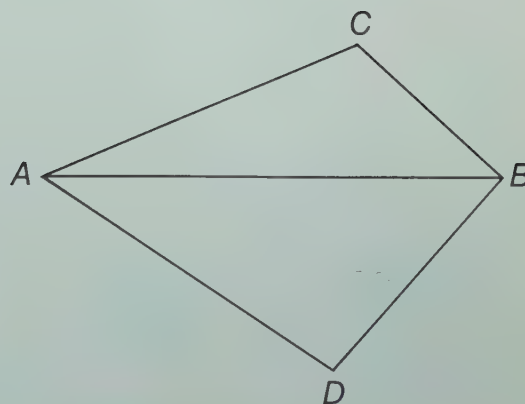
2. This figure shows 3 angles,  $BAC$ ,  $BAD$ , and  $CAD$ . Draw another ray from point  $A$ . Put a point  $E$  on your ray. Name as many "new" angles as you can.

$\angle EAD$      $\angle EAC$      $\angle EAB$



3. How many angles can you find and name on this figure?

$\angle CAB$      $\angle CAD$      $\angle BAD$      $\angle ABD$   
 $\angle ABC$      $\angle DBC$      $\angle ADB$      $\angle ACB$



Don't insist that the children find all of the segments and angles. Do encourage them to find as many as they possibly can.

## ● Parallel Lines

1. The symbol for parallel is  $\parallel$ .

We write

$\ell \parallel m$



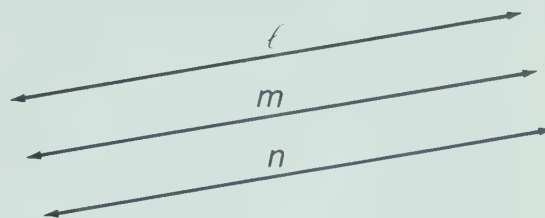
and read

$\ell$  is parallel to  $m$

What other pairs of parallel lines can you find and record in the picture?

$\ell \parallel n$

$m \parallel n$



2. How many pairs of parallel lines can you name in this figure?

$a \parallel b$

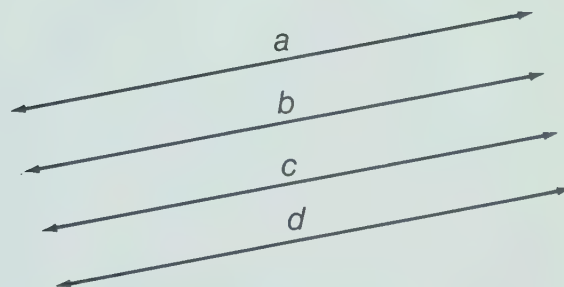
$a \parallel c$

$a \parallel d$

$b \parallel c$

$b \parallel d$

$c \parallel d$



3. How many pairs of parallel edges can you find on the cube? Three of them are given below.

$AB \parallel EF$

$AB \parallel HG$

$AB \parallel DC$

$EF \parallel HG$

$EF \parallel DC$

$HG \parallel DC$

$AE \parallel DH$

$AE \parallel CG$

$AE \parallel BF$

$DH \parallel CG$

$DH \parallel BF$

$CG \parallel BF$

$AD \parallel EH$

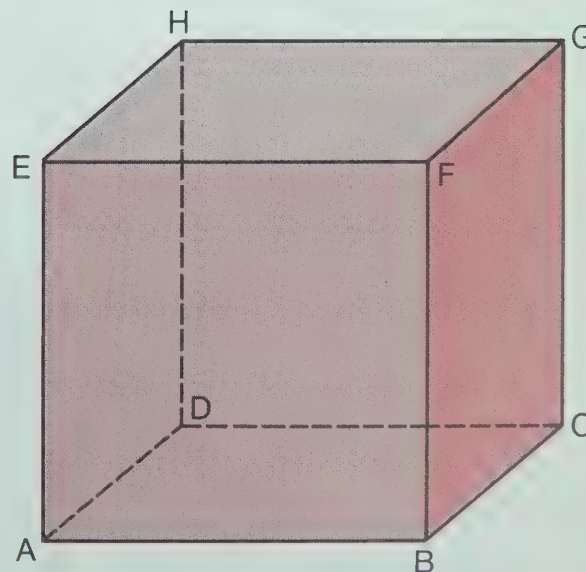
$AD \parallel FG$

$AD \parallel BC$

$EH \parallel FG$

$EH \parallel BC$

$FG \parallel BC$

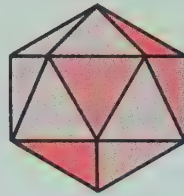


It will be helpful to have the children examine a cube to understand the idea of parallel edges prior to finding as many of them as they can in exercise 3.



## ●Space Figures

Here are two interesting geometric figures. You can make them with the patterns shown below. Can you give the number of faces, edges, and vertices of each figure?

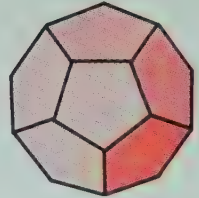


Icosahedron

Faces 20

Edges 30

Vertices 12

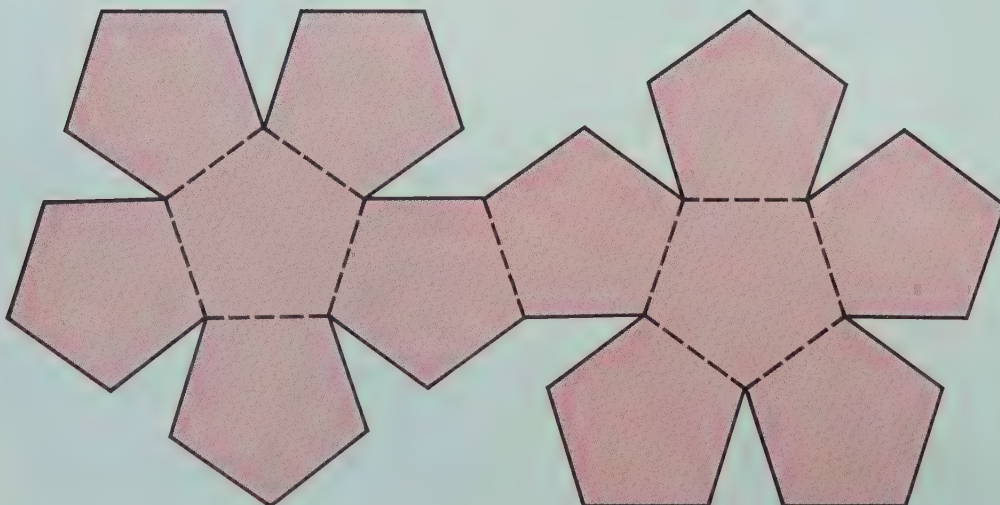
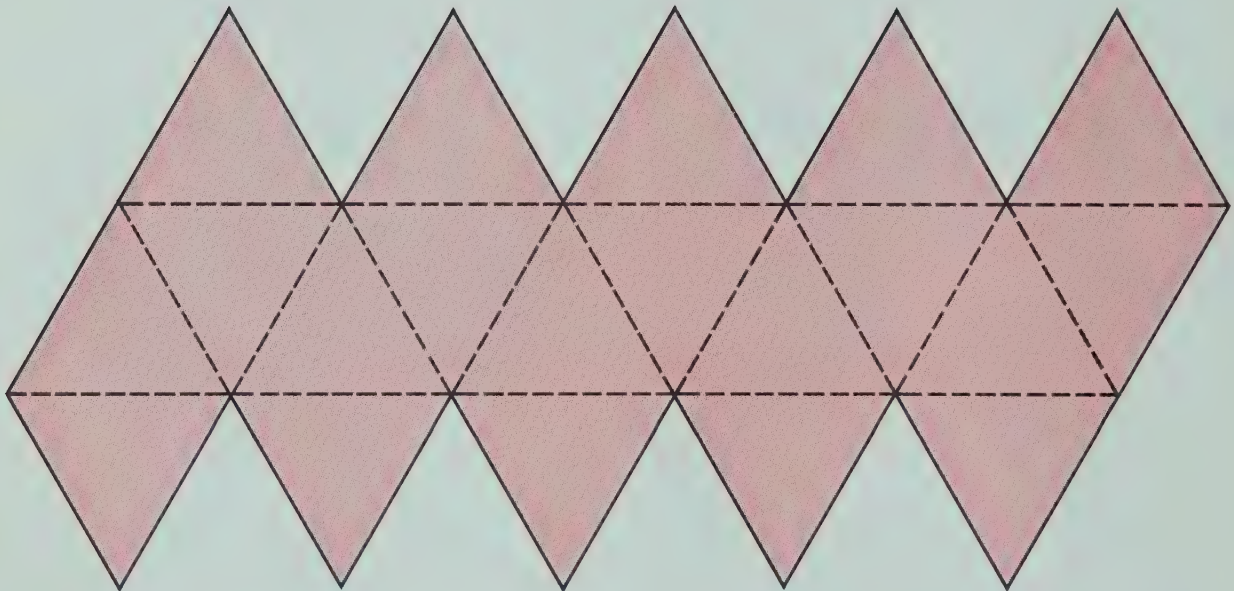


Dodecahedron

Faces 12

Edges 30

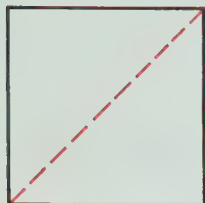
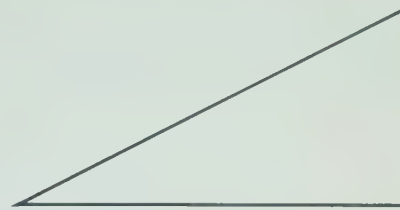
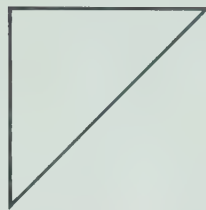
Vertices 20



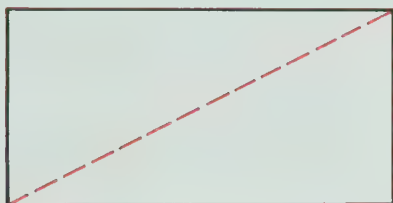
It will be instructive to have the children actually try to make at least one of these figures.

## ● Polygons

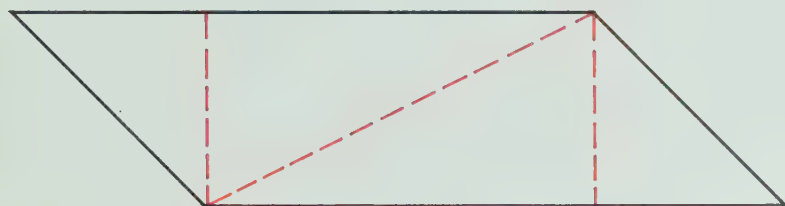
Trace and cut out two of each triangle.  
Use some or all of your triangles to make these special polygons. Draw lines inside the figures to show how you placed the triangles.



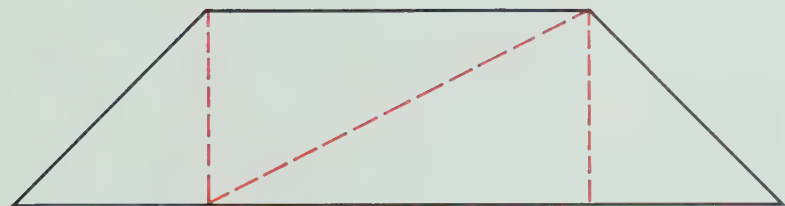
Square



Rectangle



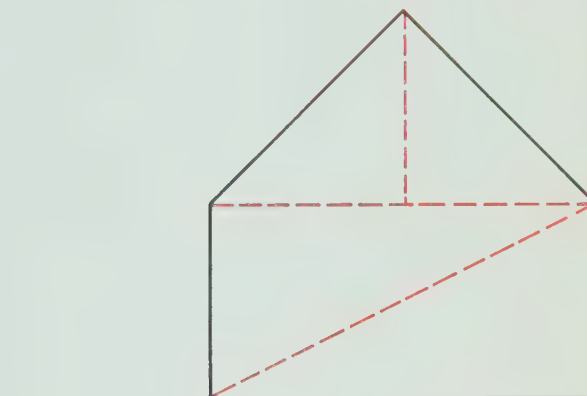
Parallelogram



Trapezoid



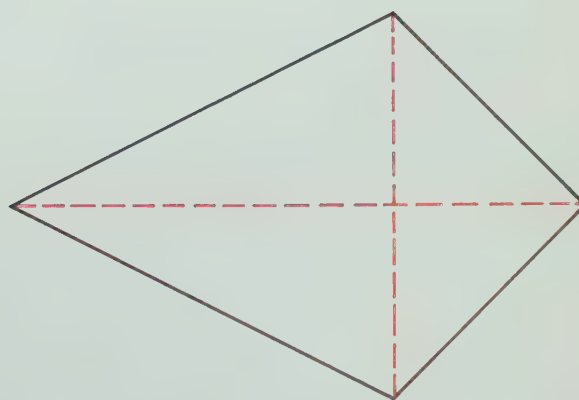
Parallelogram



Pentagon



Triangle



Quadrilateral

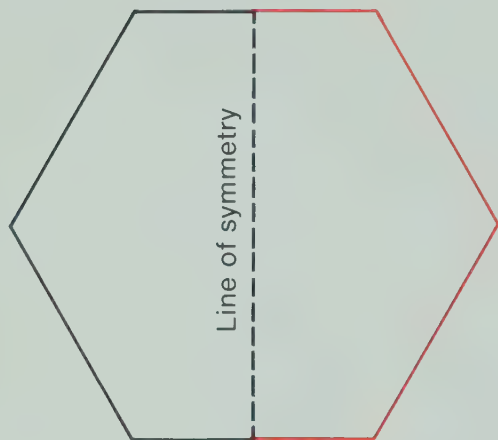
Don't stress the names of the various figures unless the children show particular interest in them.



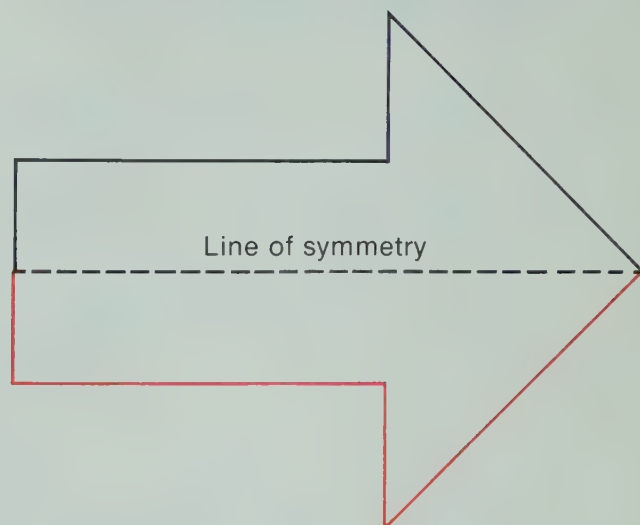
## ● Symmetric Figures

Without cutting out or folding this page, can you find a way to draw “the other half” of each figure so you have a symmetric figure.

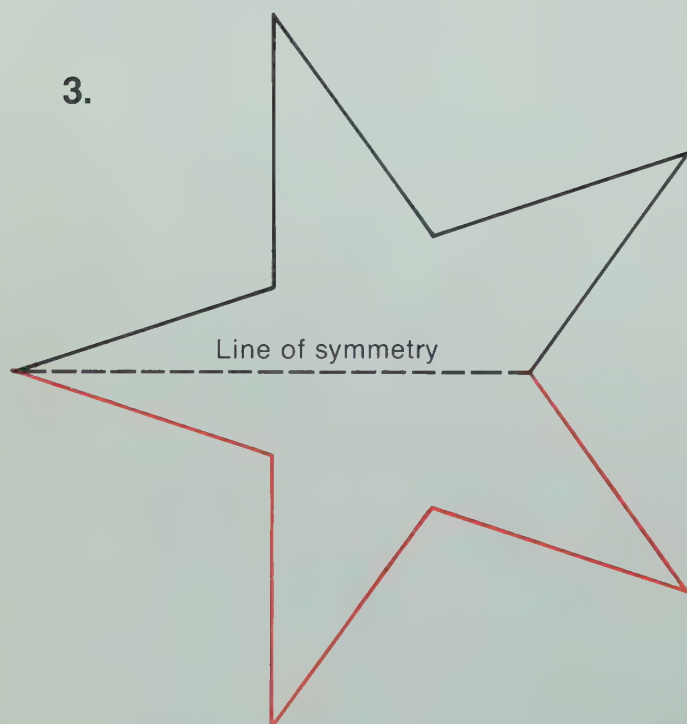
1.



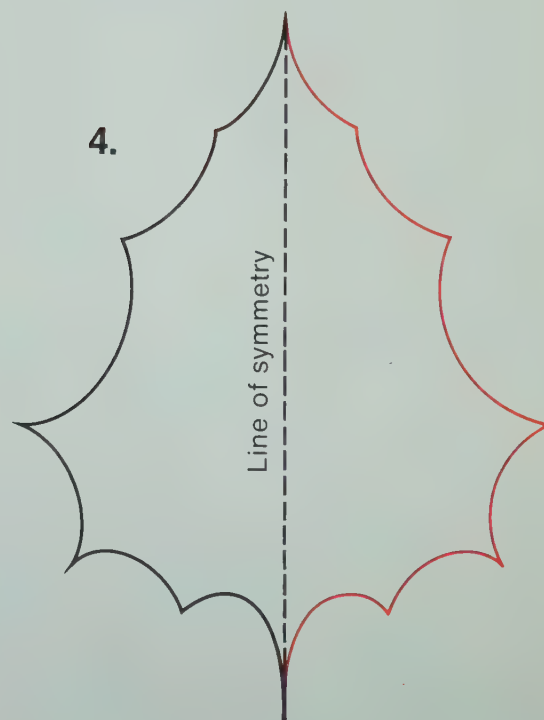
2.



3.



4.



Possible methods for solving these exercises include, using graph paper, tracing paper, measurement, ruler and compass.

1. Complete each sequence.

A 0, 2, 4, 6, 8, 10, 12, 14, 16, 18

B 0, 3, 6, 9, 12, 15, 18, 21, 24, 27

C 0, 4, 8, 12, 16, 20, 24, 28, 32, 36

D 0, 5, 10, 15, 20, 25, 30, 35, 40, 45

E 0, 6, 12, 18, 24, 30, 36, 42, 48, 54

F 0, 7, 14, 21, 28, 35, 42, 49, 56, 63

G 0, 8, 16, 24, 32, 40, 48, 56, 64, 72

H 0, 9, 18, 27, 36, 45, 54, 63, 72, 81

2. Solve the equations. The sequences above may help you.

A  $6 \times 2 = \underline{12}$

$8 \times 2 = \underline{16}$

$18 \div 2 = \underline{9}$

B  $7 \times 3 = \underline{21}$

$5 \times 3 = \underline{15}$

$24 \div 3 = \underline{8}$

C  $9 \times 4 = \underline{36}$

$6 \times 4 = \underline{24}$

$28 \div 4 = \underline{7}$

D  $8 \times 5 = \underline{40}$

$7 \times 5 = \underline{35}$

$45 \div 5 = \underline{9}$

E  $9 \times 6 = \underline{54}$

$5 \times 6 = \underline{30}$

$48 \div 6 = \underline{8}$

F  $4 \times 7 = \underline{28}$

$8 \times 7 = \underline{56}$

$63 \div 7 = \underline{9}$

G  $7 \times 8 = \underline{56}$

$9 \times 8 = \underline{72}$

$64 \div 8 = \underline{8}$

H  $6 \times 9 = \underline{54}$

$8 \times 9 = \underline{72}$

$45 \div 9 = \underline{5}$

It will be helpful if the children understand how the sequences relate to the multiplication exercises. For example you might have a child explain how 6 twos relate to skip counting by twos.



## ● Using Logic to Find Products

1. Use the table to help you find these products.

A  $8 \times 8286 = \underline{66,288}$

B  $6 \times 8286 = \underline{49,716}$

C  $4 \times 7654 = \underline{30,616}$

D  $6 \times 7654 = \underline{45,924}$

E  $7 \times 9847 = \underline{68,929}$

F  $5 \times 9847 = \underline{49,235}$

G  $3 \times 7385 = \underline{22,155}$

H  $5 \times 7385 = \underline{36,925}$

$7 \times 8286 = 58,002$

$5 \times 7654 = 38,270$

$6 \times 9847 = 59,082$

$4 \times 7385 = 29,540$

$8 \times 3485 = 27,880$

$6 \times 7856 = 47,136$

I  $9 \times 3485 = \underline{31,365}$

J  $7 \times 3485 = \underline{24,395}$

K  $5 \times 7856 = \underline{39,280}$

L  $7 \times 7856 = \underline{54,992}$

2. Use this table to help you find these products.

A  $8 \times 786 = \underline{6288}$

B  $6 \times 786 = \underline{4716}$

C  $14 \times 786 = \underline{11,004}$

D  $12 \times 786 = \underline{9432}$

E  $9 \times 963 = \underline{8667}$

F  $11 \times 963 = \underline{10,593}$

G  $12 \times 963 = \underline{11,556}$

H  $8 \times 963 = \underline{7704}$

I  $15 \times 3468 = \underline{52,020}$

J  $17 \times 3468 = \underline{58,956}$

K  $16 \times 3468 = \underline{55,488}$

$3 \times 786 = 2358$

$5 \times 786 = 3930$

$9 \times 786 = 7074$

$7 \times 3468 = 24,276$

$8 \times 3468 = 27,744$

$9 \times 3468 = 31,212$

$4 \times 963 = 3852$

$5 \times 963 = 4815$

$7 \times 963 = 6741$

$12 \times 638 = 7656$

$13 \times 638 = 8294$

$16 \times 638 = 10,208$

L  $18 \times 3468 = \underline{62,424}$

M  $25 \times 638 = \underline{15,950}$

N  $28 \times 638 = \underline{17,864}$

O  $29 \times 638 = \underline{18,502}$

P  $32 \times 638 = \underline{20,416}$

In exercise 1 it may be helpful to point out to children how in 1A they can find eight times 8286 by adding 8286 to 58002. In exercise 2A, it may be helpful to point out that they can find eight times 786 by adding the two products in the table.

## ● Multiplication Facts

Complete the table.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

1. Which products are in the table just

one time? 1, 25, 49, 64, 81

2. Some of the products are in the table exactly 3 times. How many of them

can you find? 4, 9, 16, 36

3. No product other than zero appears more than 4 times. How many products can you find that appear

4 times? 6, 8, 12, 18, 24

4. Choose one of your numbers from exercise 3. How many different multiplication equations can you write for your number? **Examples**

$$2 \times 6 = 12$$

$$6 \times 2 = 12$$

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$1 \times 12 = 12$$

$$12 \times 1 = 12$$

Observe that the products which are in the table 1 or 3 times are the squares of the whole numbers 1 through 9. In exercise 4, note that the child may write equations that do not appear in the table for his number. For example, if the number 12 is chosen, the equation  $1 \times 12 = 12$  and  $12 \times 1 = 12$  could be written.



## ● Multiplication Tables

1. Here is a multiplication chart with the numbers mixed up. Can you give the missing products?

×	8	2	5	9
6	48	12	30	54
4	32	8	20	36
7	56	14	35	63
3	24	6	15	27

2. These multiplication tables have both missing products and missing factors. Can you find the missing numbers in each one?

**A**

×	5	9	3	7
4	20	36	12	28
6	30	54	18	42
2	10	18	6	14
4	20	36	12	28

**B**

×	9	5	1	8
4	36	20	4	32
6	54	30	6	48
2	18	10	2	16
3	27	15	3	24

**C**

×	6	6	7	4
3	18	18	21	12
1	6	6	7	4
2	12	12	14	8
5	30	30	35	20

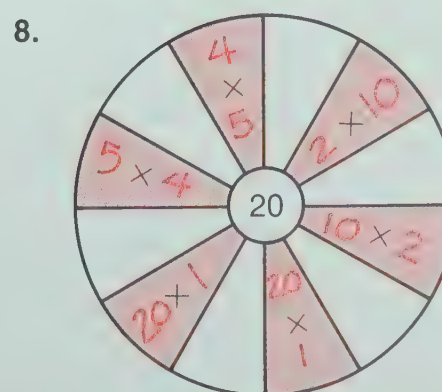
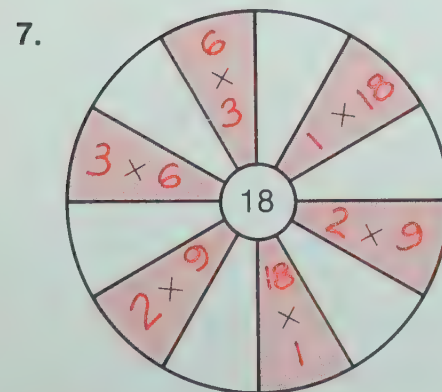
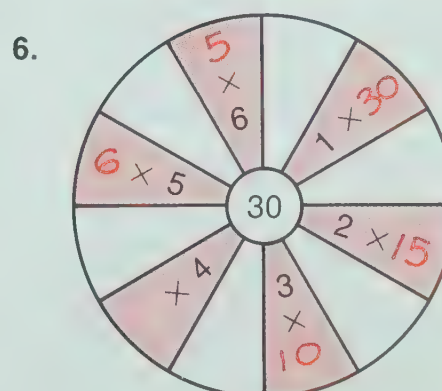
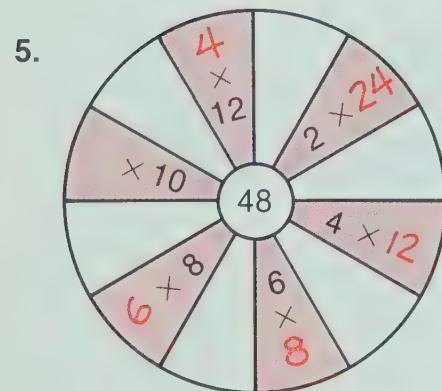
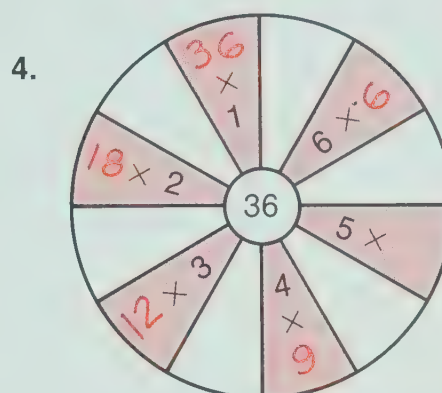
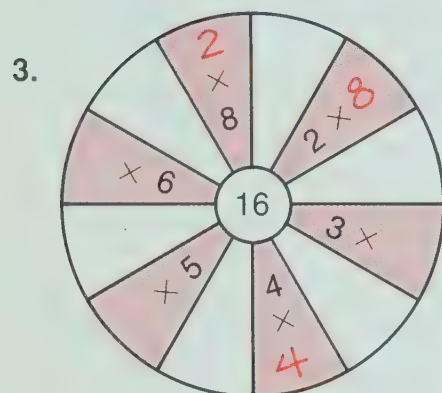
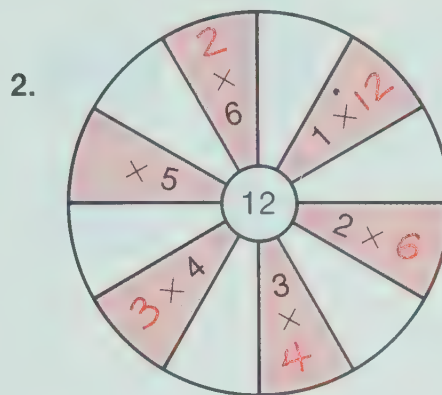
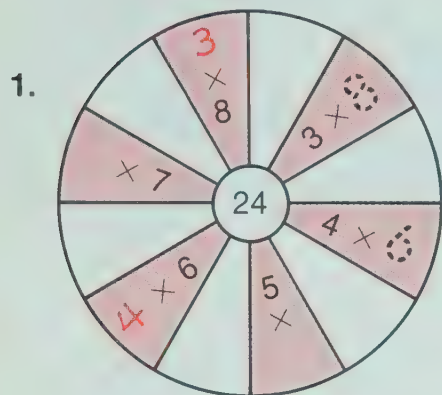
**D**

×	1	6	2	4
7	7	42	14	28
8	8	48	16	32
3	3	18	6	12
9	9	54	18	36

In order to solve exercise 2, the child must find both missing products and missing factors. This can be done by a series of logical reasonings where certain missing factors must be found prior to finding other missing products.

## Finding Missing Factors

The number in the center should be the product of the two numbers on the spokes. Give as many missing spoke numbers as you can find. Some of the spokes can't be completed with whole numbers.



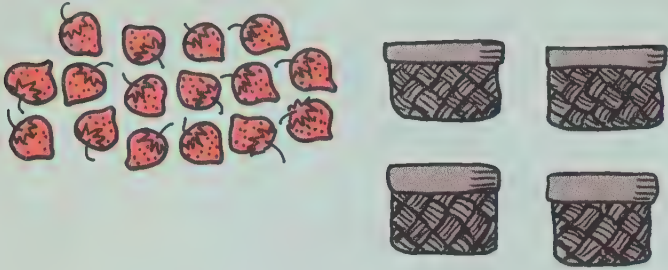
In some of the exercises there are no whole number answers for the missing factors. These should be left blank unless the child is advanced enough to give answers involving fractions.



## ● Division and Sets

Write and solve a short story problem for each picture.

1.

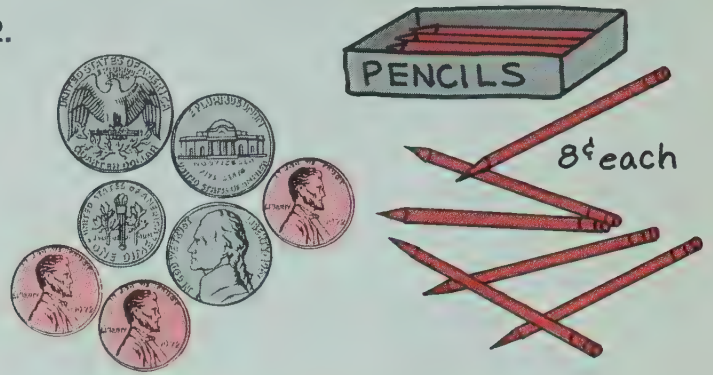


Have 16 strawberries.

How many can you put  
in each bag if they are  
equally divided?

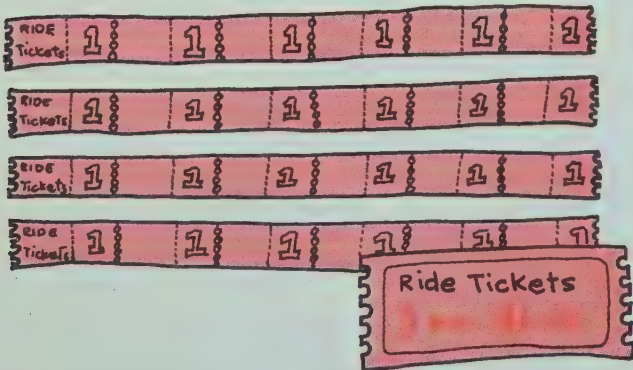
$$16 \div 4 = 4$$

2.



How many 8¢ pencils can  
you buy if you have 48¢?  
 $48 \div 8 = 6$

3.



Want to buy 24 ride tickets.  
If the tickets are 3 for \$1.00,  
what is the total cost?  
 $24 \div 3 = 8$  or \$8.00

4.



18 cookies are to be  
shared equally among 3  
children. How many cookies  
will each child receive?  
 $18 \div 3 = 6$

The attempt here is to get the child to write division problems from the information or the suggestion given in the picture. The answers given are only samples of possible problems that can be created.

## ● Problem Solving



1. There were 17 boys and 18 girls going on the picnic. If 5 children ride in each car, how many cars are needed? 7

$$\begin{array}{r} 7 \\ 5 \overline{)35} \end{array}$$

2. Put numbers in this problem that make sense. Then solve your problem.  
*answers will vary*  
 Tam had \_\_\_\_\_ photographs. She put \_\_\_\_\_ on each page of her album.  
 How many pages did she use? \_\_\_\_\_

3. Make up a multiplication problem using these numbers: 4 and 7. Then solve your problem.

*answers will vary*

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4. Write a division problem using numbers of your own choosing. Solve your problem.

*answers will vary*

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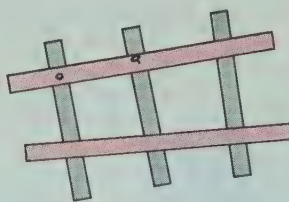
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Each problem allows the child more creativity, in terms of making his own problem, than the one before.

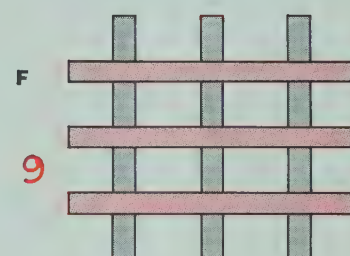
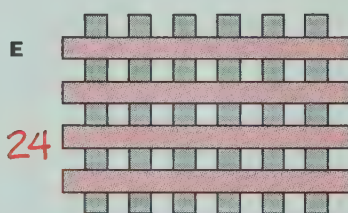
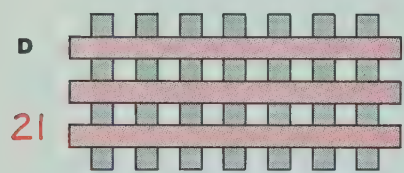
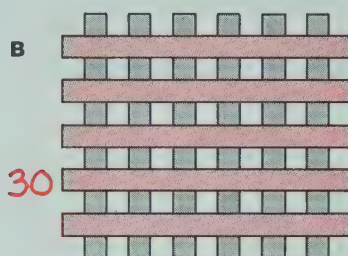
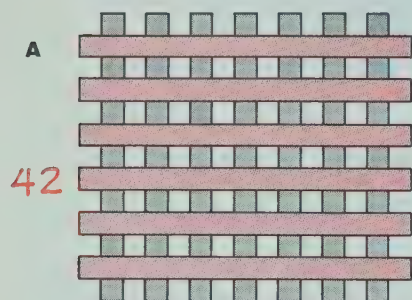


## ● Pairing and Multiplication

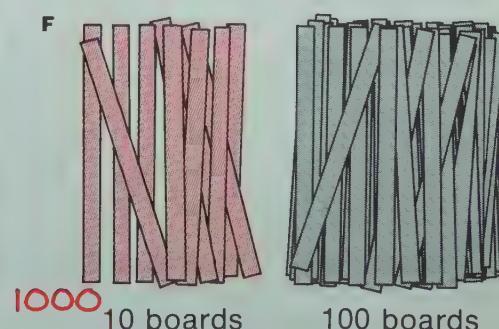
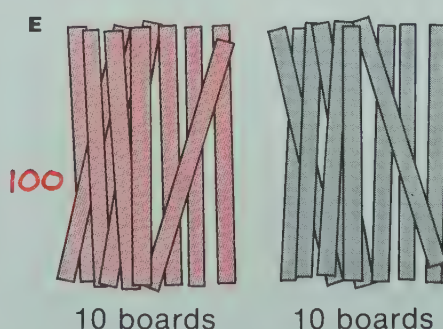
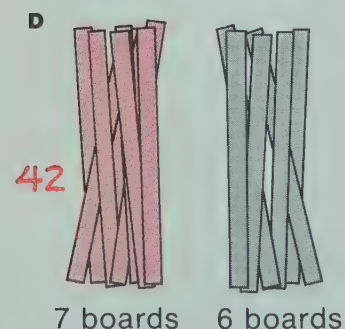
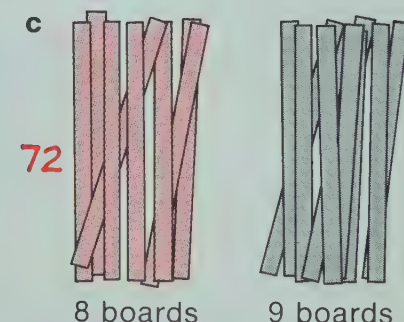
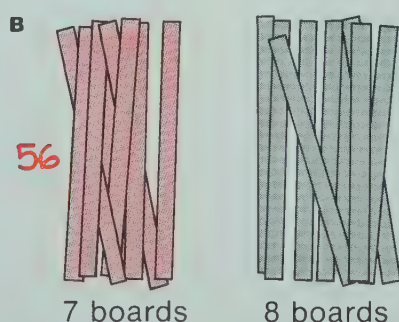
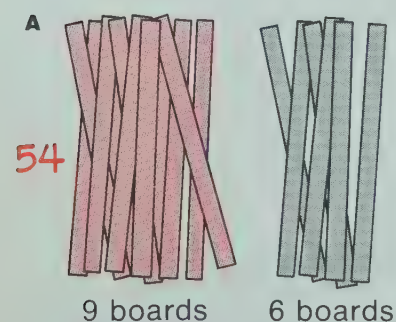
1. How many nails would it take to nail 5 boards together if you put one at each crossing? **6**



2. Give the number of nails for each of these sets of boards.



3. Give the number of nails for each of these sets of boards if they are nailed together like those above.



The concept being developed here is that of the relationship between pairing and multiplication. After children have an opportunity to give the number of nails in the various exercises, encourage them to observe how the concept of multiplication fits in with the number of nails and the number of boards in each of the two sets.

1. Find the products.

A  $5 \times 70 = \underline{350}$

F  $7 \times 80 = \underline{560}$

B  $5 \times 700 = \underline{3500}$

G  $7 \times 800 = \underline{5600}$

C  $5 \times 7000 = \underline{35,000}$

H  $7 \times 8000 = \underline{56,000}$

D  $5 \times 70,000 = \underline{350,000}$

I  $7 \times 80,000 = \underline{560,000}$

E  $5 \times 700,000 = \underline{3,500,000}$

J  $7 \times 800,000 = \underline{5,600,000}$

2. Solve the equations.

A  $6 \times 8000 = n$   
 $n = \underline{48,000}$

B  $5 \times 3000 = n$   
 $n = \underline{15,000}$

C  $4 \times 9000 = n$   
 $n = \underline{36,000}$

D  $9 \times 30,000 = n$   
 $n = \underline{270,000}$

E  $8 \times 40,000 = n$   
 $n = \underline{320,000}$

F  $7 \times 90,000 = n$   
 $n = \underline{630,000}$

G  $6 \times 500,000 = n$   
 $n = \underline{3,000,000}$

H  $3 \times 700,000 = n$   
 $n = \underline{2,100,000}$

I  $8 \times 200,000 = n$   
 $n = \underline{1,600,000}$

3. Use the table to help you solve these equations.

$10^2 = 100$

$10^4 = 10,000$

$10^3 = 1000$

$10^5 = 100,000$

A  $6 \times 10^3 = \underline{6000}$

D  $9 \times 10^4 = \underline{90,000}$

G  $7 \times 10^3 = \underline{7000}$

B  $7 \times 10^2 = \underline{700}$

E  $4 \times 10^2 = \underline{400}$

H  $6 \times 10^4 = \underline{60,000}$

C  $3 \times 10^5 = \underline{300,000}$

F  $8 \times 10^5 = \underline{800,000}$

I  $5 \times 10^5 = \underline{500,000}$

4. Solve the equations.

A  $6 \times 4 \times 10^4 = \underline{240,000}$

E  $9 \times 2 \times 10^3 = \underline{18,000}$

I  $7 \times 8 \times 10^2 = \underline{5600}$

B  $4 \times 7 \times 10^2 = \underline{2800}$

F  $6 \times 5 \times 10^5 = \underline{3,000,000}$

J  $8 \times 6 \times 10^5 = \underline{4,800,000}$

C  $7 \times 6 \times 10^3 = \underline{42,000}$

G  $2 \times 8 \times 10^4 = \underline{160,000}$

K  $3 \times 7 \times 10^3 = \underline{21,000}$

D  $8 \times 5 \times 10^5 = \underline{4,000,000}$

H  $5 \times 9 \times 10^3 = \underline{45,000}$

L  $9 \times 3 \times 10^4 = \underline{270,000}$

This lesson is designed to help children understand the relationship between the number of zeros in the factors and the number of zeros in the products.



## ● Finding Missing Factors

1. One of the factors in each equation should end in zero.  
Can you write six different equations for each product?

**A** 120  $\underline{1 \times 120} = 120$   
 $\underline{3 \times 40} = 120$

$\underline{6 \times 20} = 120$   
 $\underline{12 \times 10} = 120$

$\underline{4 \times 30} = 120$   
 $\underline{2 \times 60} = 120$

**B** 180  $\underline{1 \times 180} = 180$   
 $\underline{2 \times 90} = 180$

$\underline{3 \times 60} = 180$   
 $\underline{18 \times 10} = 180$

$\underline{6 \times 30} = 180$   
 $\underline{9 \times 20} = 180$

**C** 240  $\underline{1 \times 240} = 240$   
 $\underline{3 \times 80} = 240$

$\underline{6 \times 40} = 240$   
 $\underline{24 \times 10} = 240$

$\underline{4 \times 60} = 240$   
 $\underline{8 \times 30} = 240$

2. Solve the equations.

**A**  $4 \times n \times 10^3 = 24,000$   
 $n = \underline{6}$

**B**  $n \times 8 \times 10^4 = 480,000$   
 $n = \underline{6}$

**C**  $2 \times n \times 10^4 = 180,000$   
 $n = \underline{9}$

**D**  $n \times 8 \times 10^2 = 3200$   
 $n = \underline{4}$

**E**  $6 \times n \times 10^2 = 1200$   
 $n = \underline{2}$

**F**  $n \times 4 \times 10^3 = 20,000$   
 $n = \underline{5}$

**G**  $3 \times n \times 10^3 = 21,000$   
 $n = \underline{7}$

**H**  $n \times 3 \times 10^5 = 2,100,000$   
 $n = \underline{7}$

**I**  $5 \times n \times 10^5 = 3,000,000$   
 $n = \underline{6}$

3. When multiples of ten are expressed as  $10^2$ ,  $10^3$ ,  $10^4$ , and  $10^5$ , they are called **powers of ten**. Give the correct power of ten for each equation.

**A**  $5 \times 7 \times n = 3500$   
 $n = \underline{10^2}$

**B**  $4 \times 8 \times n = 32,000$   
 $n = \underline{10^3}$

**C**  $7 \times 6 \times n = 420,000$   
 $n = \underline{10^4}$

**D**  $5 \times 9 \times n = 45,000$   
 $n = \underline{10^3}$

**E**  $8 \times 9 \times n = 7200$   
 $n = \underline{10^2}$

**F**  $6 \times 4 \times n = 2,400,000$   
 $n = \underline{10^5}$

This lesson is designed to help children understand the relationship between the number of zeros in the factors and the number of zeros in the products.

## ● Finding Special Quotients

1. Give the correct power of ten for each equation.

**A**  $3500 \div 5 = 7 \times n$

$n = 10^2$

**B**  $24,000 \div 6 = 4 \times n$

$n = 10^3$

**C**  $4200 \div 7 = 6 \times n$

$n = 10^2$

**D**  $28,000 \div 4 = 7 \times n$

$n = 10^3$

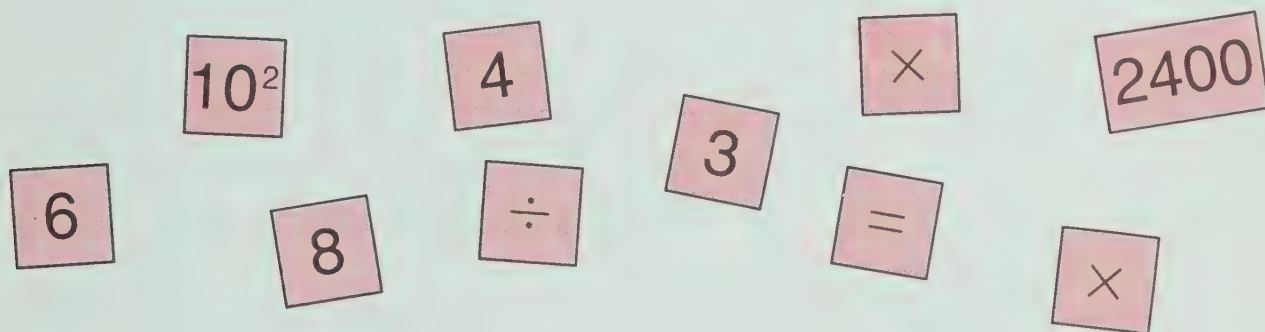
**E**  $180,000 \div 6 = 3 \times n$

$n = 10^4$

**F**  $27,000 \div 3 = 9 \times n$

$n = 10^3$

2. How many different equations can you write using the symbols on these cards?



EXAMPLES:

$$4 \times 6 \times 10^2 = 2400$$

or

$$2400 \div 8 = 3 \times 10^2$$

Sample answers:

$$2400 \div 6 = 4 \times 10^2$$

$$3 \times 8 \times 10^2 = 2400$$

$$6 \times 10^2 = 2400 \div 4$$

$$4 \times 10^2 = 2400 \div 6$$

$$8 \times 10^2 = 2400 \div 3$$

$$3 \times 10^2 = 2400 \div 8$$

These exercises are designed to help children use powers of ten in special products and quotients.



## ● Factors with Zeros

Put in some zeros so the equations are correct and different.

Sample answers:

1.  $3\text{ } \underline{\text{00}} \times 8\text{ } \underline{\hspace{1cm}} = 2400$

$3\text{ } \underline{\text{0}} \times 8\text{ } \underline{\text{0}} = 2400$

$3\text{ } \underline{\hspace{1cm}} \times 8\text{ } \underline{\text{00}} = 2400$

2.  $7\text{ } \underline{\text{00}} \times 4\text{ } \underline{\hspace{1cm}} = 2800$

$7\text{ } \underline{\text{0}} \times 4\text{ } \underline{\text{0}} = 2800$

$7\text{ } \underline{\hspace{1cm}} \times 4\text{ } \underline{\text{00}} = 2800$

3.  $5\text{ } \underline{\text{00}} \times 6\text{ } \underline{\hspace{1cm}} = 3000$

$5\text{ } \underline{\text{0}} \times 6\text{ } \underline{\text{0}} = 3000$

$5\text{ } \underline{\hspace{1cm}} \times 6\text{ } \underline{\text{00}} = 3000$

4.  $9\text{ } \underline{\text{00}} \times 7\text{ } \underline{\hspace{1cm}} = 6300$

$9\text{ } \underline{\text{0}} \times 7\text{ } \underline{\text{0}} = 6300$

$9\text{ } \underline{\hspace{1cm}} \times 7\text{ } \underline{\text{00}} = 6300$

5.  $6\text{ } \underline{\text{000}} \times 8\text{ } \underline{\hspace{1cm}} = 48,000$

$6\text{ } \underline{\text{00}} \times 8\text{ } \underline{\text{0}} = 48,000$

$6\text{ } \underline{\text{0}} \times 8\text{ } \underline{\text{00}} = 48,000$

6.  $8\text{ } \underline{\text{00}} \times 5\text{ } \underline{\text{0}} = 40,000$

$8\text{ } \underline{\text{0}} \times 5\text{ } \underline{\text{00}} = 40,000$

$8\text{ } \underline{\text{000}} \times 5\text{ } \underline{\hspace{1cm}} = 40,000$

7.  $3\text{ } \underline{\text{000}} \times 9\text{ } \underline{\text{0}} = 270,000$

$3\text{ } \underline{\text{0}} \times 9\text{ } \underline{\text{000}} = 270,000$

$3\text{ } \underline{\text{00}} \times 9\text{ } \underline{\text{00}} = 270,000$

8.  $5\text{ } \underline{\text{000}} \times 4\text{ } \underline{\text{0}} = 200,000$

$5\text{ } \underline{\text{0}} \times 4\text{ } \underline{\text{000}} = 200,000$

$5\text{ } \underline{\text{00}} \times 4\text{ } \underline{\text{00}} = 200,000$

9.  $9\text{ } \underline{\text{0}} \times 6\text{ } \underline{\hspace{1cm}} = 54\text{ } \underline{\text{0}}$

$9\text{ } \underline{\text{00}} \times 6\text{ } \underline{\hspace{1cm}} = 54\text{ } \underline{\text{00}}$

$9\text{ } \underline{\text{0}} \times 6\text{ } \underline{\text{0}} = 54\text{ } \underline{\text{00}}$

$9\text{ } \underline{\text{00}} \times 6\text{ } \underline{\text{0}} = 54\text{ } \underline{\text{000}}$

$9\text{ } \underline{\text{0}} \times 6\text{ } \underline{\text{00}} = 54\text{ } \underline{\text{000}}$

10.  $5\text{ } \underline{\text{0}} \times 2\text{ } \underline{\text{0}} = 1\text{ } \underline{\text{000}}$

$5\text{ } \underline{\text{00}} \times 2\text{ } \underline{\text{0}} = 1\text{ } \underline{\text{0,000}}$

$5\text{ } \underline{\text{0}} \times 2\text{ } \underline{\text{00}} = 1\text{ } \underline{\text{0,000}}$

$5\text{ } \underline{\text{00}} \times 2\text{ } \underline{\text{00}} = 1\text{ } \underline{\text{00,000}}$

$5\text{ } \underline{\text{000}} \times 2\text{ } \underline{\text{00}} = 1\text{ } \underline{\text{,000,000}}$

This lesson is designed to help children understand the relationship between the number of zeros in the factors and the number of zeros in the products.

## ● Finding Special Quotients

1. Fill in the missing numbers in each multiplication table.

**A**

×	100	300	600	1000	4000	5000
4	400	1200	2400	4000	16,000	20,000
6	600	1800	3600	6000	24,000	30,000
5	500	1500	3000	5000	20,000	25,000
7	700	2100	4200	7000	28,000	35,000

**B**

×	20	40	30	60	80	50
40	800	1600	1200	2400	3200	2000
70	1400	2800	2100	4200	5600	3500
20	400	800	600	1200	1600	1000
60	1200	2400	1800	3600	4800	3000

**C**

×	100	300	400	200	800	500
50	5000	15,000	20,000	10,000	40,000	25,000
80	8000	24,000	32,000	16,000	64,000	40,000
30	3000	9000	12,000	6,000	24,000	15,000
90	9000	27,000	36,000	18,000	72,000	45,000

2. Put in some zeros so the equations are different and correct. *Order of answers may vary.*

**A**  $1400 \div 7 \underline{0} = 2 \underline{0}$

**B**  $3200 \div 4 \underline{0} = 8 \underline{0}$

$1400 \div 7 \underline{00} = 2 \underline{\quad}$

$3200 \div 4 \underline{00} = 8 \underline{\quad}$

$1400 \div 7 \underline{\quad} = 2 \underline{00}$

$3200 \div 4 \underline{\quad} = 8 \underline{00}$

**C**  $4000 \div 5 \underline{0} = 8 \underline{0}$

**D**  $7200 \div 8 \underline{0} = 9 \underline{0}$

$4000 \div 5 \underline{00} = 8 \underline{\quad}$

$7200 \div 8 \underline{00} = 9 \underline{\quad}$

$4000 \div 5 \underline{\quad} = 8 \underline{00}$

$7200 \div 8 \underline{\quad} = 9 \underline{00}$

**E**  $45,000 \div 9 \underline{000} = 5 \underline{\quad}$

**F**  $30,000 \div 5 \underline{000} = 6 \underline{\quad}$

$45,000 \div 9 \underline{00} = 5 \underline{0}$

$30,000 \div 5 \underline{00} = 6 \underline{0}$

$45,000 \div 9 \underline{0} = 5 \underline{00}$

$30,000 \div 5 \underline{0} = 6 \underline{00}$

$45,000 \div 9 \underline{\quad} = 5 \underline{000}$

$30,000 \div 5 \underline{\quad} = 6 \underline{000}$

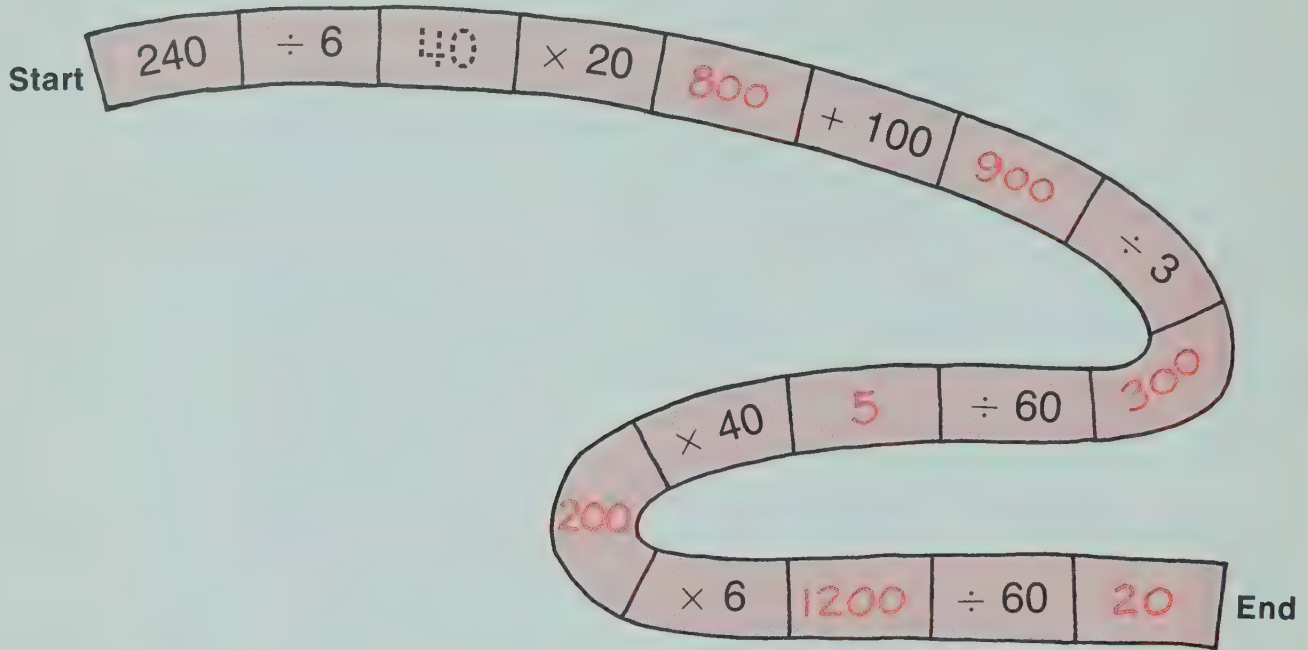
The tables in these exercises are designed to help children work with special quotients. Enough information is given in each table for the children to find the factors in the top row of the table.



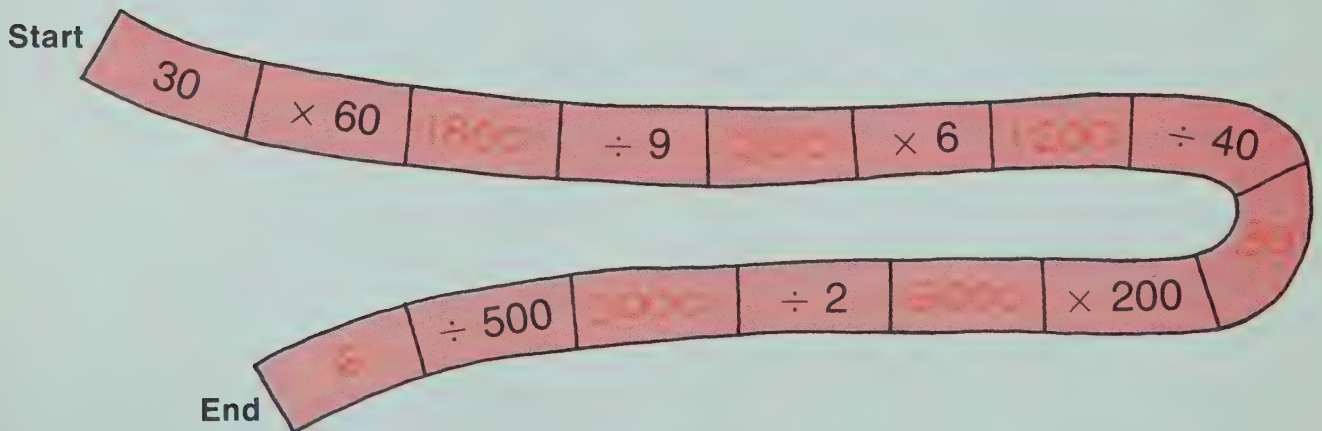
## ● Finding Special Products and Quotients

Fill in the missing numbers. If your work is correct you should end with the number shown in red.

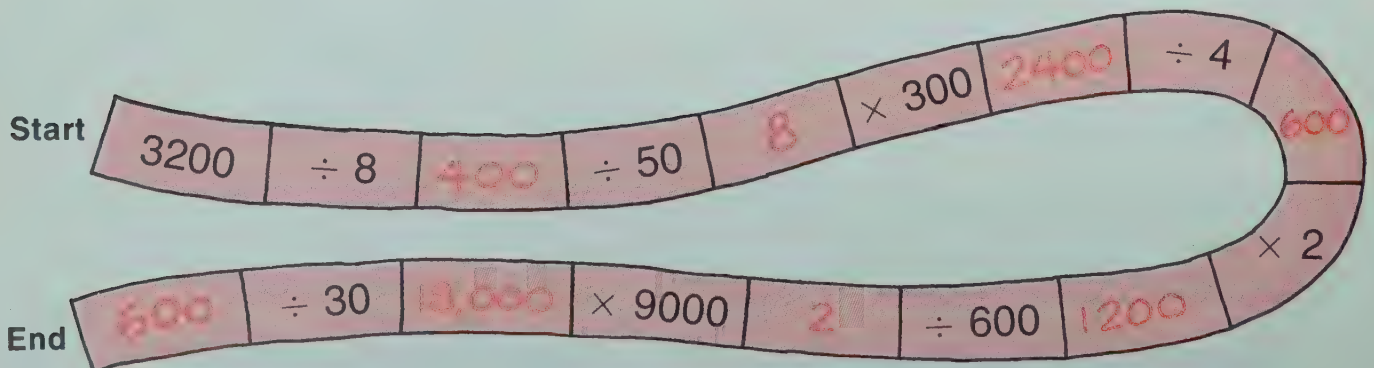
1.



2.



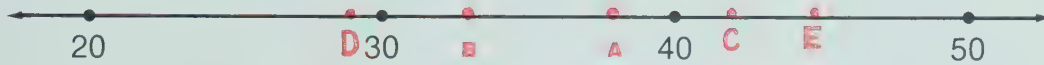
3.



On each number line, estimate the position of the numbers with dots as in the examples for **A** and **B**. Put the letter under each dot.

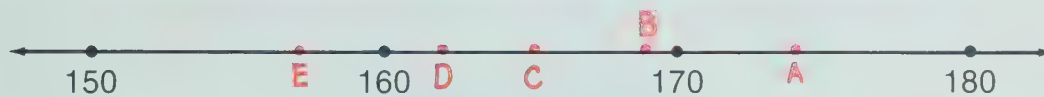
1.

**A** 38      **B** 33      **C** 42      **D** 29      **E** 35



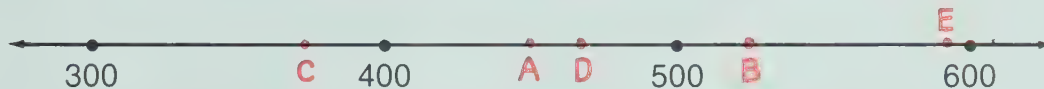
2.

**A** 174      **B** 169      **C** 165      **D** 162      **E** 157



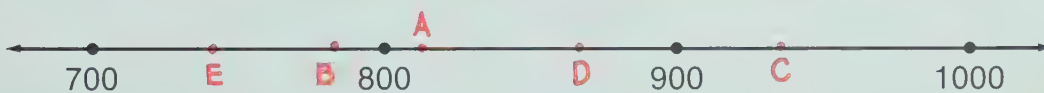
3.

**A** 450      **B** 525      **C** 375      **D** 467      **E** 592



4.

**A** 812      **B** 783      **C** 937      **D** 869      **E** 744



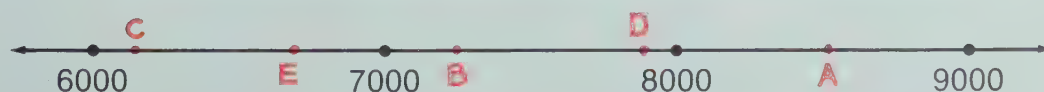
5.

**A** 1850      **B** 1925      **C** 1775      **D** 1733      **E** 1989



6.

**A** 8512      **B** 7248      **C** 6124      **D** 7896      **E** 6698



These exercises are designed to help children see which of two numbers (that are multiples of 10, 100, or 1000) a given number is closer to without specific division marks on the number line.

## ● Estimating Totals

1. You have this much money. Use estimation to decide which of these totals you'll be able to pay. Write "yes" or "no" on the paper.



**A**

\$	1.98
	1.89
	1.88

total: **NO**

**\$5.75**

**B**

\$	.99
	.98
	.95
	.89
	.99

total: **YES**

**\$4.80**

**C**

\$	1.12
	.95
	1.10
	.98

total: **YES**

**\$4.15**

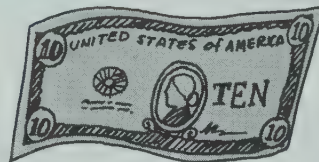
**D**

\$	2.95
	.89
	1.98

total: **NO**

**\$5.82**

2. Which of these will you be able to pay with the \$10 bill?



**A**

\$	2.98
	3.95
	1.99
	.96

total: **YES**

**\$9.88**

**B**

\$	3.79
	4.88
	2.49
	1.10

total: **NO**

**\$12.26**

**C**

\$	1.95
	1.89
	1.99
	1.88
	1.98

total: **YES**

**\$9.69**

**D**

\$	.89
	4.98
	1.00
	2.95
	.95

total: **NO**

**\$10.77**

3. Which of these will you be able to pay with the \$20 bill?



**A**

\$	4.95
	4.95
	4.95
	4.95

total: **YES**

**\$19.80**

**B**

\$	5.98
	4.98
	5.98
	4.98

total: **NO**

**\$21.92**

**C**

\$	7.00
	3.95
	2.98
	6.95

total: **NO**

**\$20.88**

**D**

\$	3.79
	2.89
	2.95
	2.99
	2.98
	2.99

total: **YES**

**\$18.59**

Encourage the children to use mental arithmetic and estimation to make decisions on these exercises. If they make mistakes in the exercises, have them check by actually doing the arithmetic.



## ● Estimating Products

1. Fill in the estimation tables.

**A**

×	9 9	1 9 8	3 9 5	6 1 2	9 9 8	1 0 2 1
3	3 0 0	6 0 0	12 0 0	18 0 0	3 0 0 0	3 0 0 0
4	4 0 0	8 0 0	16 0 0	24 0 0	4 0 0 0	4 0 0 0
6	6 0 0	12 0 0	24 0 0	36 0 0	6 0 0 0	6 0 0 0
8	8 0 0	16 0 0	32 0 0	48 0 0	8 0 0 0	8 0 0 0

**B**

×	3 1	3 9	2 2	4 7	7 1	8 9
2 8	9 0 0	12 0 0	6 0 0	15 0 0	21 0 0	27 0 0
1 9	6 0 0	8 0 0	4 0 0	10 0 0	14 0 0	18 0 0
4 2	12 0 0	16 0 0	8 0 0	20 0 0	28 0 0	36 0 0
5 8	18 0 0	24 0 0	12 0 0	30 0 0	42 0 0	54 0 0

**C**

×	9 8	3 1 2	1 9 5	4 8 7	2 0 8	6 9 9
3 2	3 0 0 0	9 0 0 0	6 0 0 0	15,000	6 0 0 0	21,000
1 8	2 0 0 0	6 0 0 0	4 0 0 0	10,000	4 0 0 0	14,000
4 1	4 0 0 0	12,000	8 0 0 0	20,000	8 0 0 0	28,000
6 9	7 0 0 0	21,000	14,000	35,000	14,000	49,000

2. Put in some numbers of your own so the estimates in the table are "close." Then complete the tables.

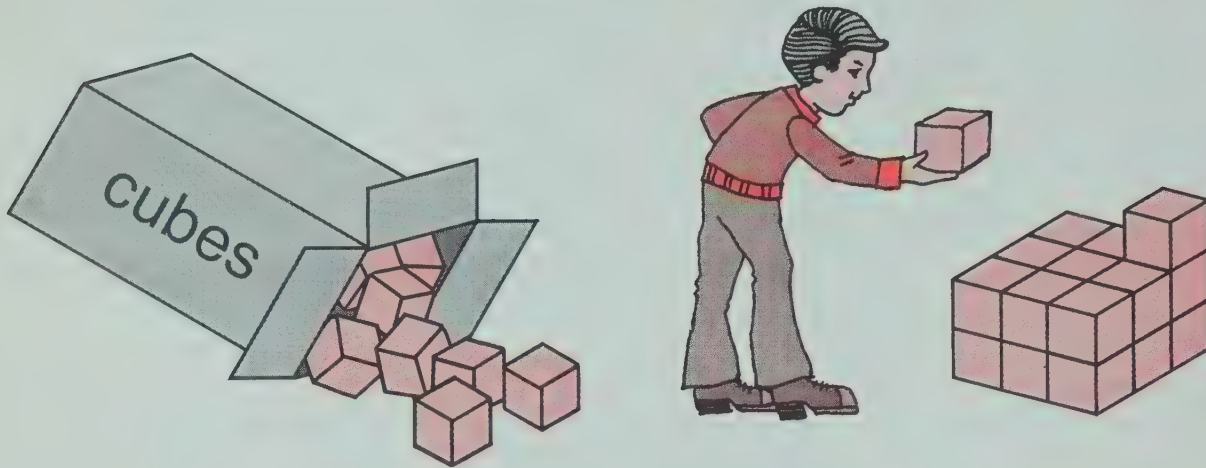
any number close to

×	20	50	40	60	100	300
2	40	1 0 0	80	1 2 0	200	6 0 0
7	1 4 0	350	2 8 0	4 2 0	700	2100
2 9	600	1 5 0 0	1200	1800	3 0 0 0	9000
5 2	1 0 0 0	2500	2 0 0 0	3000	5 0 0 0	15,000

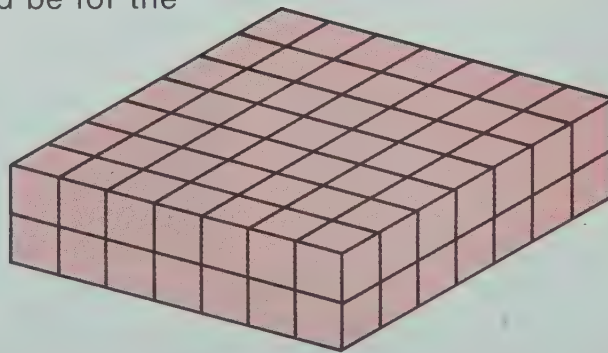
Be sure the children understand that in completing these tables they are working with estimates involving multiples of 10 or 100. Enough examples are given that they should see the kind of numbers required in the estimation tables.

## ● Estimating Quotients

1. Estimate about how many cubes tall this “building” would be for the different numbers of cubes.



- A 178 cubes. About how tall? 20
- B 631 cubes. About how tall? 70
- C 358 cubes. About how tall? 40
- D 542 cubes. About how tall? 60
2. Estimate how tall this “building” would be for the different numbers of cubes.

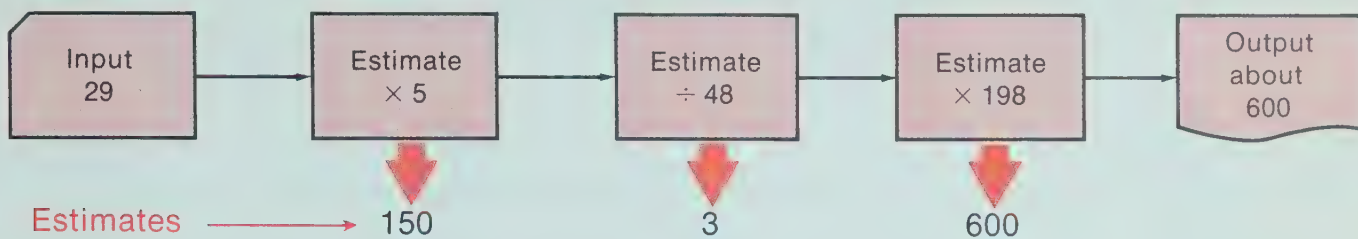


- A 298 cubes. About how tall? 6
- B 410 cubes. About how tall? 8
- C 1498 cubes. About how tall? 30
- D 3018 cubes. About how tall? 60

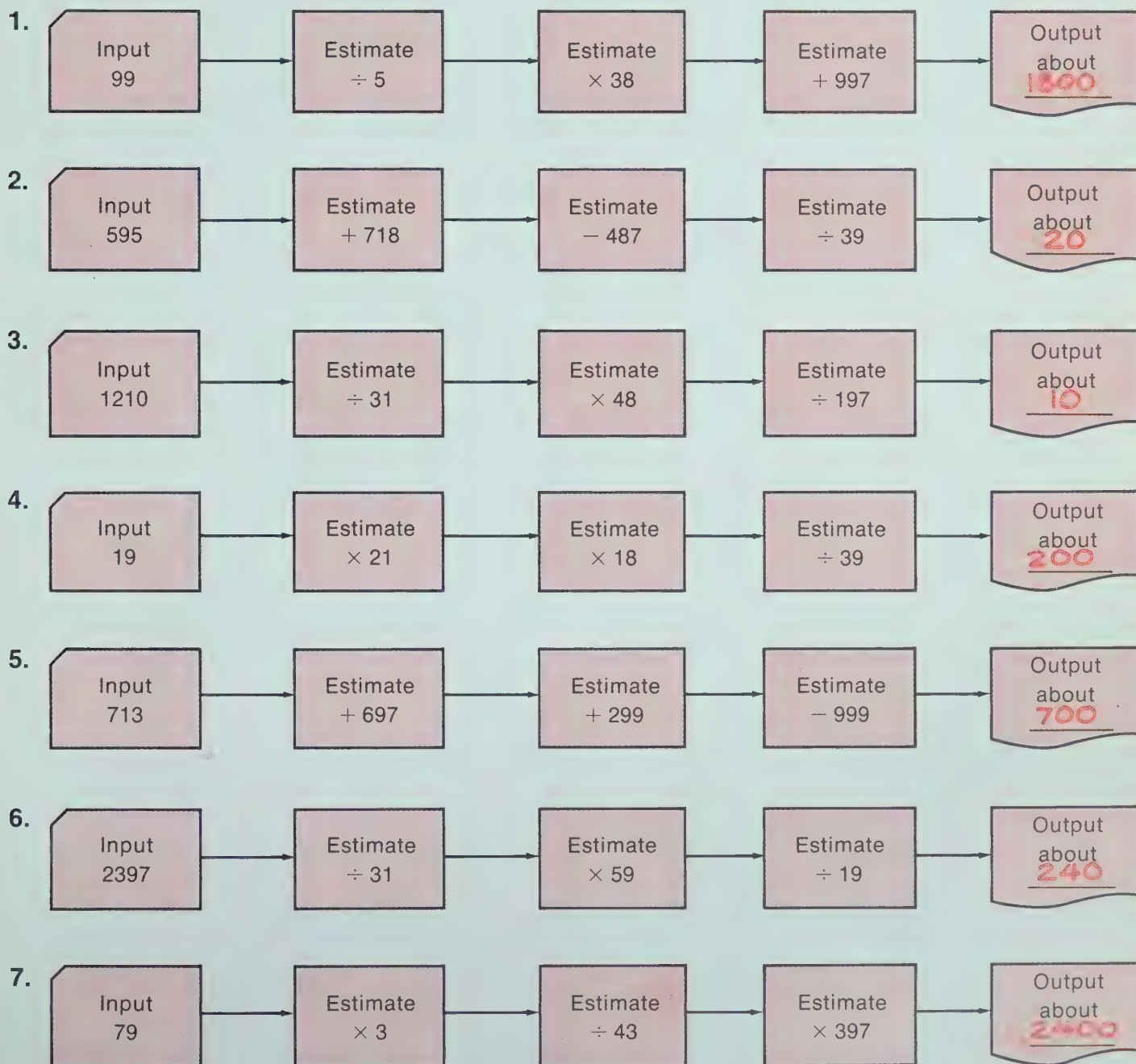
In solving these problems the children should recognize that in the building for exercise 1 each layer is 9 and the building for exercise 2 each layer is 49.

## ● Estimating

Study the flow chart examples using estimates.



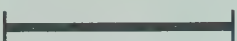
Find each output number using estimates.



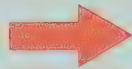
One of the major purposes of this lesson is to provide experiences in mental arithmetic involving estimates and work with multiples of 10, 100, and 1000.



## ● Estimation for Fun

1. 

If this length is  
79 units,



then this length is

about 200 units.

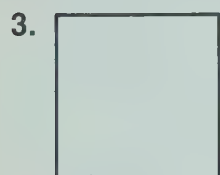
2. 

If this length is  
19 units,



then this length is

about 100 units.



If this area is  
49 units,



then this area is

about 200 units.

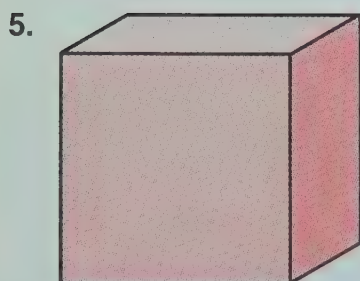


If this area is  
21 units,

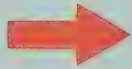


then this  
area is about

180 units.

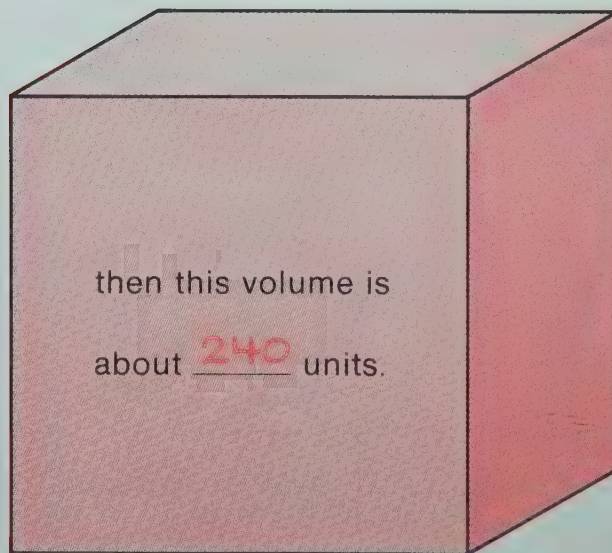


If this volume is  
29 units,



then this volume is

about 240 units.



Note that each given measurement is a number that is easily rounded to a multiple of ten.

1. Grade Lisa's paper.

1. $\begin{array}{r} 38 \\ \times 4 \\ \hline 152 \end{array}$	2. $\begin{array}{r} 57 \\ \times 2 \\ \hline 114 \end{array}$	3. $\begin{array}{r} 46 \\ \times 8 \\ \hline 366 \end{array}$	4. $\begin{array}{r} 92 \\ \times 7 \\ \hline 644 \end{array}$	5. <i>LISA</i> $\begin{array}{r} 28 \\ \times 6 \\ \hline 166 \end{array}$
6. $\begin{array}{r} 49 \\ \times 4 \\ \hline 196 \end{array}$	7. $\begin{array}{r} 83 \\ \times 6 \\ \hline 478 \end{array}$	8. $\begin{array}{r} 85 \\ \times 4 \\ \hline 340 \end{array}$	9. $\begin{array}{r} 64 \\ \times 8 \\ \hline 492 \end{array}$	10. $\begin{array}{r} 18 \\ \times 6 \\ \hline 106 \end{array}$

2. Each mistake Lisa made was for the same reason. Can you tell Lisa what she is doing wrong?

*Lisa thinks that  $6 \times 8 = 46$*

3. Grade Don's paper.

1. $\begin{array}{r} 62 \\ \times 3 \\ \hline 186 \end{array}$	2. $\begin{array}{r} 73 \\ \times 2 \\ \hline 146 \end{array}$	3. $\begin{array}{r} 56 \\ \times 4 \\ \hline 204 \end{array}$	4. $\begin{array}{r} 94 \\ \times 2 \\ \hline 188 \end{array}$	5. <i>Don</i> $\begin{array}{r} 47 \\ \times 3 \\ \hline 121 \end{array}$
6. $\begin{array}{r} 82 \\ \times 2 \\ \hline 164 \end{array}$	7. $\begin{array}{r} 43 \\ \times 3 \\ \hline 129 \end{array}$	8. $\begin{array}{r} 65 \\ \times 7 \\ \hline 425 \end{array}$	9. $\begin{array}{r} 42 \\ \times 3 \\ \hline 126 \end{array}$	10. $\begin{array}{r} 74 \\ \times 6 \\ \hline 424 \end{array}$

4. Can you tell Don what he is doing wrong?

*Don forgets to add the number he has regrouped in the tens column.*

## ● Multiplying with a 3-Digit Factor

1. First estimate each product. Then find the product. Find the difference between your estimate and the correct product.

**A** 
$$\begin{array}{r} 376 \\ \times 4 \\ \hline 1600 \\ \text{estimate} \\ 1504 \\ \text{product} \\ 96 \\ \text{difference} \end{array}$$

**B** 
$$\begin{array}{r} 483 \\ \times 6 \\ \hline 3000 \\ \text{estimate} \\ 2898 \\ \text{product} \\ 102 \\ \text{difference} \end{array}$$

**C** 
$$\begin{array}{r} 725 \\ \times 3 \\ \hline 2100 \\ \text{estimate} \\ 2175 \\ \text{product} \\ 75 \\ \text{difference} \end{array}$$

**D** 
$$\begin{array}{r} 321 \\ \times 9 \\ \hline 2700 \\ \text{estimate} \\ 2889 \\ \text{product} \\ 189 \\ \text{difference} \end{array}$$

**E** 
$$\begin{array}{r} 489 \\ \times 5 \\ \hline 2500 \\ \text{estimate} \\ 2445 \\ \text{product} \\ 55 \\ \text{difference} \end{array}$$

**F** 
$$\begin{array}{r} 7264 \\ \times 5 \\ \hline 35,000 \\ \text{estimate} \\ 36,320 \\ \text{product} \\ 1320 \\ \text{difference} \end{array}$$

**G** 
$$\begin{array}{r} 5438 \\ \times 3 \\ \hline 15,000 \\ \text{estimate} \\ 16,314 \\ \text{product} \\ 1314 \\ \text{difference} \end{array}$$

**H** 
$$\begin{array}{r} 6073 \\ \times 6 \\ \hline 36,000 \\ \text{estimate} \\ 36438 \\ \text{product} \\ 438 \\ \text{difference} \end{array}$$

**I** 
$$\begin{array}{r} 9802 \\ \times 4 \\ \hline 40,000 \\ \text{estimate} \\ 39,208 \\ \text{product} \\ 792 \\ \text{difference} \end{array}$$

**J** 
$$\begin{array}{r} 3743 \\ \times 7 \\ \hline 28,000 \\ \text{estimate} \\ 26,201 \\ \text{product} \\ 1799 \\ \text{difference} \end{array}$$

2. Use addition only to complete the multiplication table. (No multiplying allowed.) Hint: 5 sixty-eights is the same as the sum of 2 sixty-eights and 3 sixty-eights.

×	68	237	435	6824	5196	7843
2	136	474	870	13,648	10,392	15,686
3	204	711	1305	20,472	15,588	23,529
5	340	1185	2175	34,120	25,980	39,215
7	476	1659	3045	47,768	36,372	54,901
8	544	1896	3480	54,592	41,568	62,744
9	612	2133	3915	61,416	46,764	70,587
17	1156	4029	7395	116,008	88,332	133,331

3. Find the products below. Check your answers in the table.

**A** 
$$\begin{array}{r} 68 \\ \times 7 \\ \hline 476 \end{array}$$

**B** 
$$\begin{array}{r} 237 \\ \times 9 \\ \hline 2133 \end{array}$$

**C** 
$$\begin{array}{r} 435 \\ \times 9 \\ \hline 3915 \end{array}$$

**D** 
$$\begin{array}{r} 6824 \\ \times 8 \\ \hline 54,592 \end{array}$$

**E** 
$$\begin{array}{r} 5196 \\ \times 5 \\ \hline 25,980 \end{array}$$

**F** 
$$\begin{array}{r} 7843 \\ \times 9 \\ \hline 70,587 \end{array}$$

Notice that on the occasions when the estimate is less than the correct product, the child will have to rewrite the two numbers in order to find the difference. In exercise 2 be sure the children understand the hint that is given.



## Finding Missing Digits

1. Find the missing digit.

$$\begin{array}{r} \text{A} \quad 37 \\ \times 4 \\ \hline 148 \end{array}$$

$$\begin{array}{r} \text{B} \quad 52 \\ \times 7 \\ \hline 364 \end{array}$$

$$\begin{array}{r} \text{C} \quad 864 \\ \times 5 \\ \hline 4320 \end{array}$$

$$\begin{array}{r} \text{D} \quad 352 \\ \times 8 \\ \hline 2816 \end{array}$$

$$\begin{array}{r} \text{E} \quad 349 \\ \times 3 \\ \hline 1047 \end{array}$$

$$\begin{array}{r} \text{F} \quad 7283 \\ \times 9 \\ \hline 65,547 \end{array}$$

$$\begin{array}{r} \text{G} \quad 2465 \\ \times 4 \\ \hline 9860 \end{array}$$

$$\begin{array}{r} \text{H} \quad 5374 \\ \times 5 \\ \hline 26,870 \end{array}$$

$$\begin{array}{r} \text{I} \quad 1884 \\ \times 2 \\ \hline 3768 \end{array}$$

$$\begin{array}{r} \text{J} \quad 7982 \\ \times 6 \\ \hline 47,892 \end{array}$$

2. Find the missing digits.

$$\begin{array}{r} \text{A} \quad 43 \\ \times 7 \\ \hline 301 \end{array}$$

$$\begin{array}{r} \text{B} \quad 69 \\ \times 5 \\ \hline 345 \end{array}$$

$$\begin{array}{r} \text{C} \quad 534 \\ \times 3 \\ \hline 1602 \end{array}$$

$$\begin{array}{r} \text{D} \quad 386 \\ \times 9 \\ \hline 3474 \end{array}$$

$$\begin{array}{r} \text{E} \quad 749 \\ \times 6 \\ \hline 4494 \end{array}$$

$$\begin{array}{r} \text{F} \quad 2463 \\ \times 8 \\ \hline 19,704 \end{array}$$

$$\begin{array}{r} \text{G} \quad 7581 \\ \times 4 \\ \hline 30,324 \end{array}$$

$$\begin{array}{r} \text{H} \quad 4623 \\ \times 2 \\ \hline 9246 \end{array}$$

$$\begin{array}{r} \text{I} \quad 7867 \\ \times 4 \\ \hline 31,468 \end{array}$$

$$\begin{array}{r} \text{J} \quad 5324 \\ \times 5 \\ \hline 26,620 \end{array}$$

3. Find the missing digits.

$$\begin{array}{r} \text{A} \quad 384 \\ \times 6 \\ \hline 2304 \end{array}$$

$$\begin{array}{r} \text{B} \quad 1479 \\ \times 3 \\ \hline 4437 \end{array}$$

$$\begin{array}{r} \text{C} \quad 35 \\ \times 27 \\ \hline 245 \\ 700 \\ \hline 945 \end{array}$$

$$\begin{array}{r} \text{D} \quad 46 \\ \times 63 \\ \hline 138 \\ 2760 \\ \hline 2898 \end{array}$$

$$\begin{array}{r} \text{E} \quad 74 \\ \times 56 \\ \hline 444 \\ 3700 \\ \hline 4144 \end{array}$$

$$\begin{array}{r} \text{F} \quad 328 \\ \times 35 \\ \hline 1640 \\ 9840 \\ \hline 11,480 \end{array}$$

$$\begin{array}{r} \text{G} \quad 417 \\ \times 56 \\ \hline 2502 \\ 20850 \\ \hline 23,352 \end{array}$$

$$\begin{array}{r} \text{H} \quad 652 \\ \times 83 \\ \hline 1956 \\ 52160 \\ \hline 54,116 \end{array}$$

$$\begin{array}{r} \text{I} \quad 237 \\ \times 19 \\ \hline 2133 \\ 2370 \\ \hline 4503 \end{array}$$

$$\begin{array}{r} \text{J} \quad 489 \\ \times 33 \\ \hline 1467 \\ 14670 \\ \hline 16,137 \end{array}$$

Children should attempt to use logic in finding the missing factor in exercise 1, rather than using division. For example, in 1A the number should be 4 since that is the only product with 7 that will end in 8.

## ● Finding Products — 2 and 3-Digit Factors

1. Complete rows **A**, **B**, and **C** of the table using only addition.  
Put your own factors in rows **D**, **E**, **F**, and **G** so you can complete the table using addition.

Remember, no multiplying allowed.

	×	36	58	73	346	582	724
	30	1080	1740	2190	10,380	17,460	21,720
	50	1800	2900	3650	17,300	29,100	36,200
	80	2880	4640	5840	27,680	46,560	57,920
	4	144	232	292	1384	2328	2896
	7	252	406	511	2422	4074	5068
	9	324	522	657	3114	5238	6516
<b>A</b>	54	1944	3132	3942	18,684	31,428	39,096
<b>B</b>	39	1404	2262	2847	13,494	22,698	28,236
<b>C</b>	87	3132	5046	6351	30,102	50,634	62,988
<b>D</b>	34	1224	1972	2482	11,764	19,788	24,616
<b>E</b>	57	2052	3306	4161	19,722	33,174	41,268
<b>F</b>	89	3204	5162	6497	30,794	51,798	64,436
<b>G</b>	59	2124	3422	4307	20,414	34,338	42,716

} sample answers

2. Find these products and check your answers in the table.

$$\begin{array}{r} \text{A} \quad 58 \\ \times 39 \\ \hline 2262 \end{array}$$

$$\begin{array}{r} \text{B} \quad 73 \\ \times 87 \\ \hline 6351 \end{array}$$

$$\begin{array}{r} \text{C} \quad 346 \\ \times 54 \\ \hline 18,684 \end{array}$$

$$\begin{array}{r} \text{D} \quad 582 \\ \times 87 \\ \hline 50,634 \end{array}$$

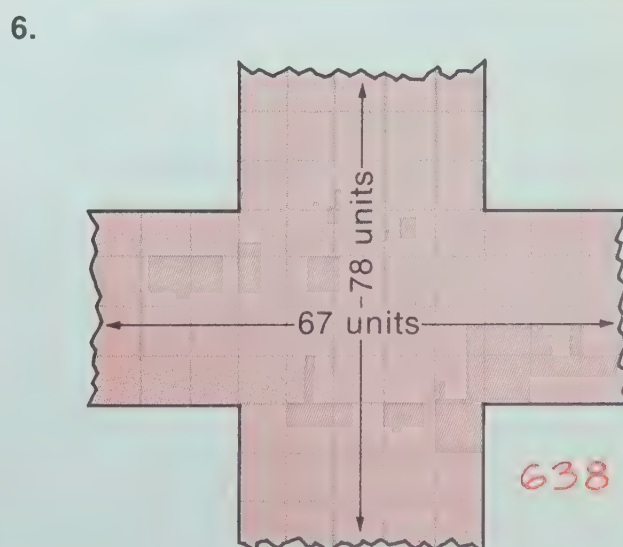
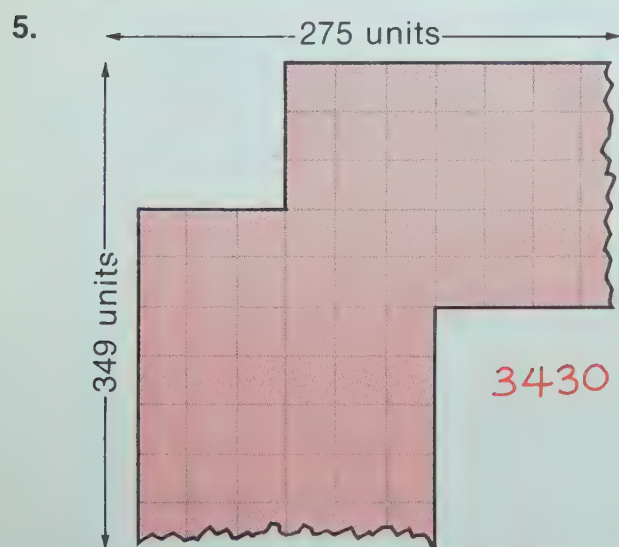
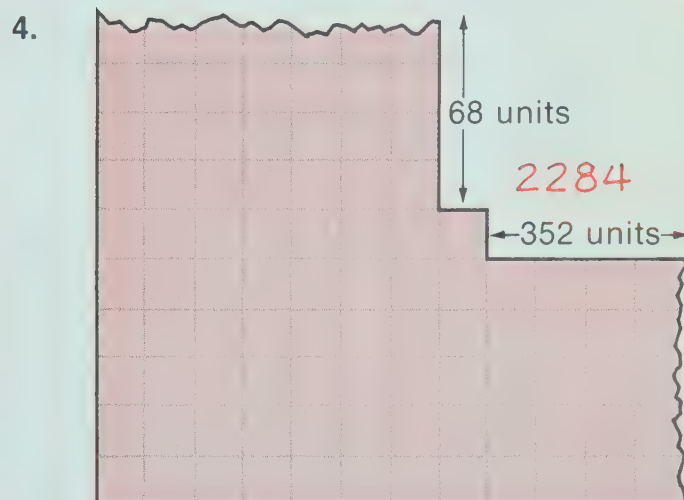
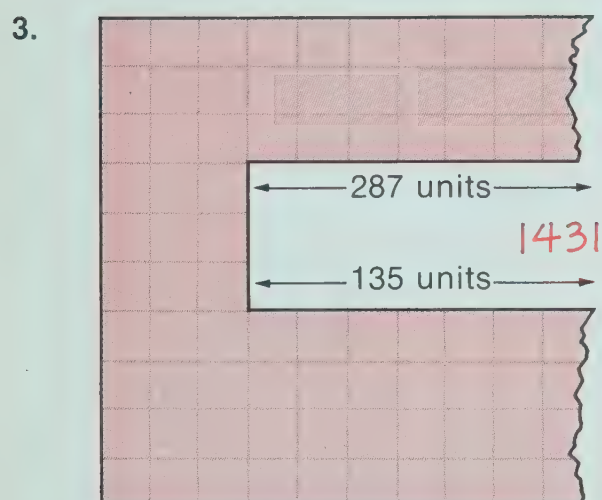
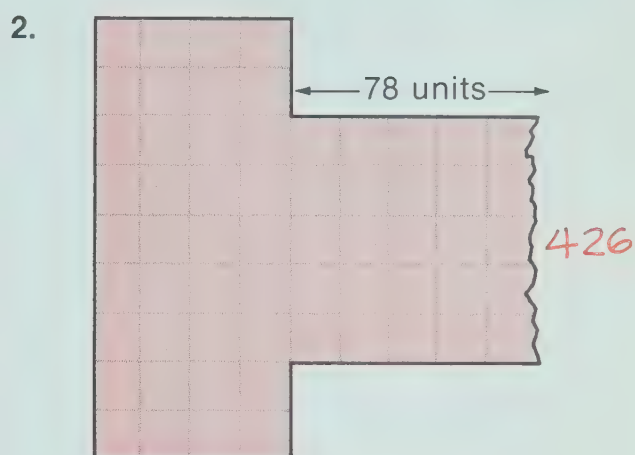
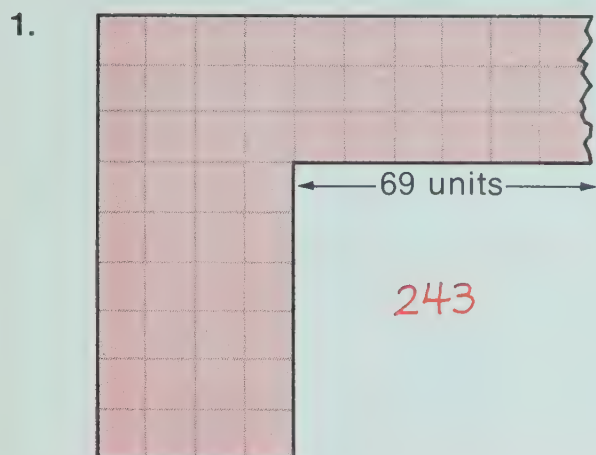
$$\begin{array}{r} \text{E} \quad 724 \\ \times 39 \\ \hline 28,236 \end{array}$$

3. Check some of your own rows in the table by multiplying.

Be sure children understand how to find a product such as  $54 \times 36$  by adding the number found in the entry for  $50 \times 36$  to the number found in the entry  $4 \times 36$ . Possibilities for D, E, F, and G are:  
34, 37, 57, 59, 84, 89.

## ● Multiplication and Measurement

Find the area of each figure.





## ● Solving Problems

1. In each problem, put in numbers of your own that make sense. Then solve your problem.

**A** If a car travels \_\_\_\_\_ kilometers each hour, how far will

it go in \_\_\_\_\_ hours?

answers will vary

**B** One summer Patty read \_\_\_\_\_ books. The average length of the book was

about \_\_\_\_\_ pages. How many pages did she read?

answers will vary

**C** The average length of a math

lesson is \_\_\_\_\_ minutes.

Judy's class had \_\_\_\_\_ lessons in November. How many minutes of math did they have in November?

answers will vary

**D** Randy's favorite sports car sells for

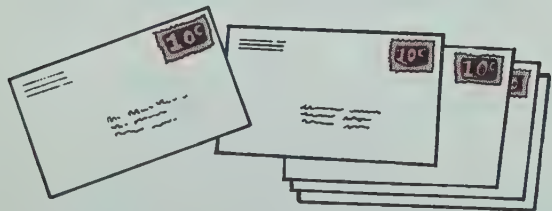
about \_\_\_\_\_ dollars. The dealer

said he sold \_\_\_\_\_ of them last year. How many dollars is this in all?

answers will vary

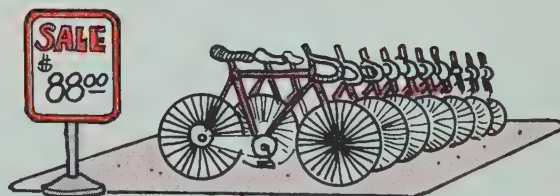
2. Write and solve a multiplication problem for each picture.

**A**



answers will vary


**B**


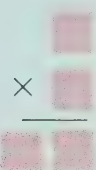
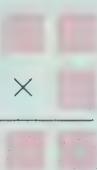
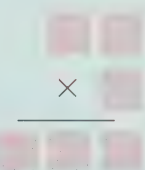

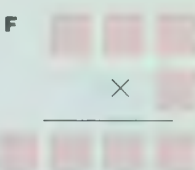



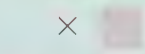

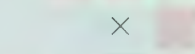








answers will vary



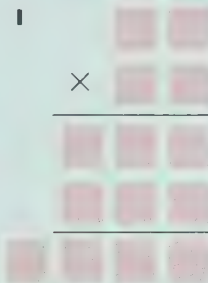
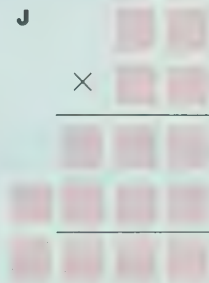

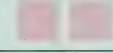
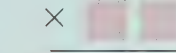
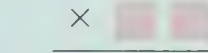
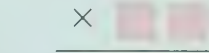
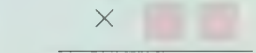







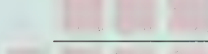
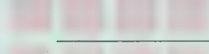
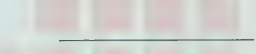




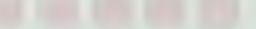
Allow as much freedom as possible in the numbers that the children use in these problems.

## ● Larger Products

- Make up your own multiplication problems so that each  has a digit and there are no extra digits.  
Have a classmate check your work. *answers will vary*
















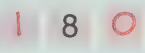


<b>A</b>		<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
$\times$		$\times$		$\times$		$\times$		$\times$		$\times$	
<hr/>											
											








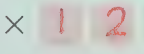



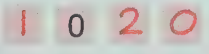
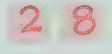

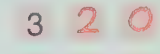
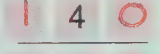





<b>G</b>		<b>H</b>		<b>I</b>		<b>J</b>		<b>K</b>	
$\times$		$\times$		$\times$		$\times$		$\times$	
<hr/>									
									
									
									

- Make your problems as you did above only be sure the given digit is correct.

*Sample answers*

<b>A</b>		<b>B</b>		<b>C</b>		<b>D</b>		<b>E</b>		<b>F</b>	
$\times$		$\times$		$\times$		$\times$		$\times$		$\times$	
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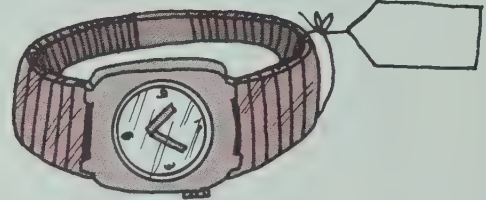
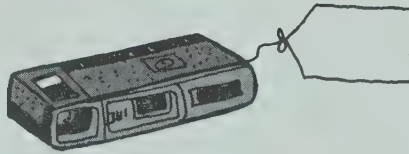
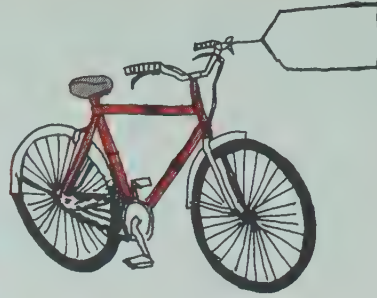
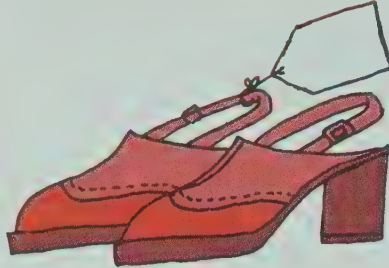
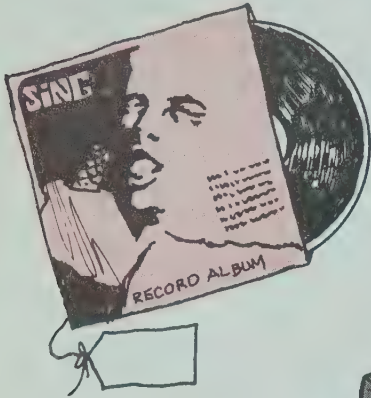
  

<b>G</b>		<b>H</b>		<b>I</b>		<b>J</b>		<b>K</b>	
$\times$		$\times$		$\times$		$\times$		$\times$	
<hr/>									
									
									
									

## ● Solving Problems

Put a price on each item that you think is about "right." Then write and solve a problem about these items that uses the operation shown.

answers will vary




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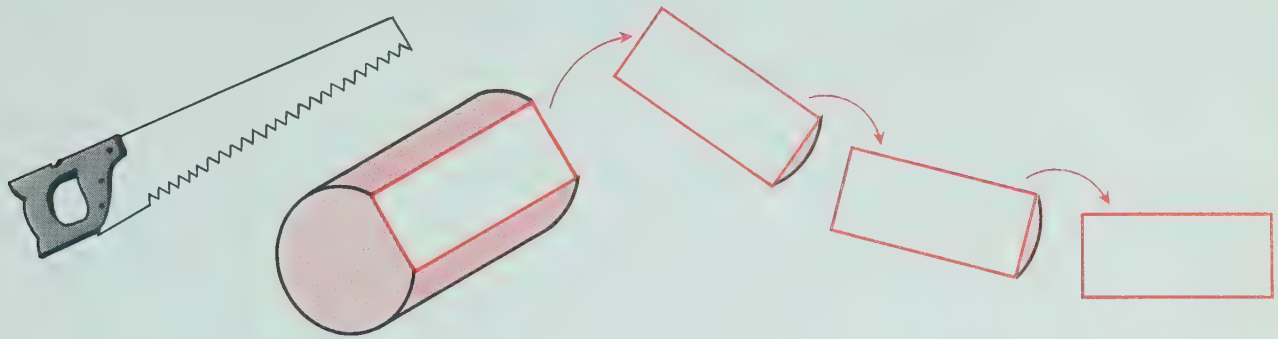
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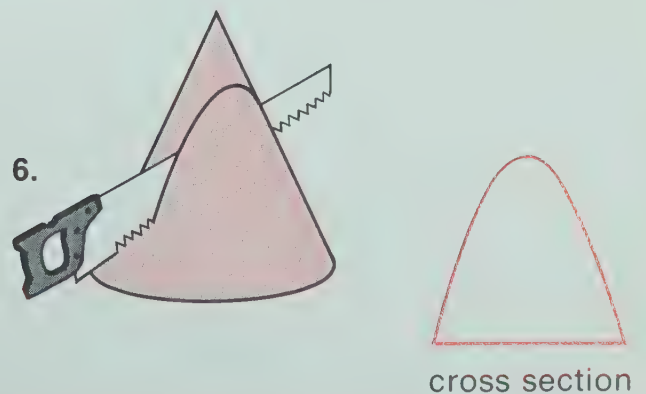
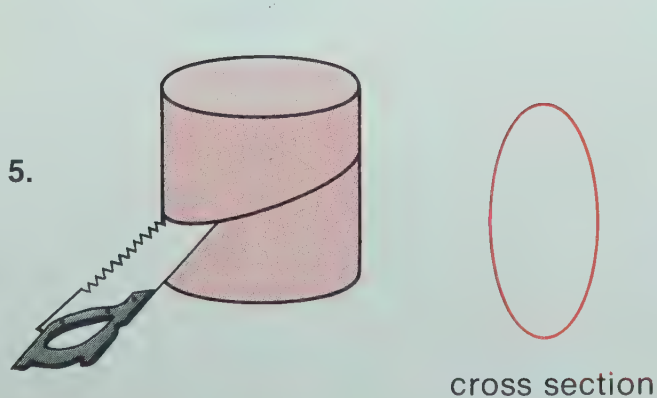
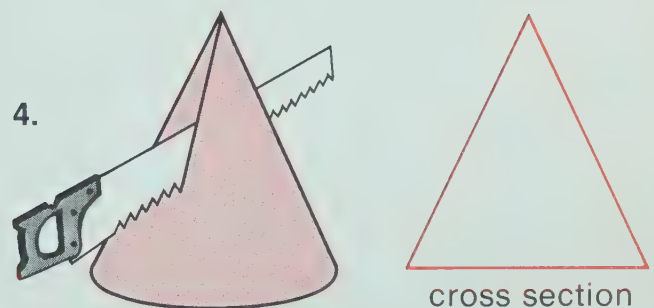
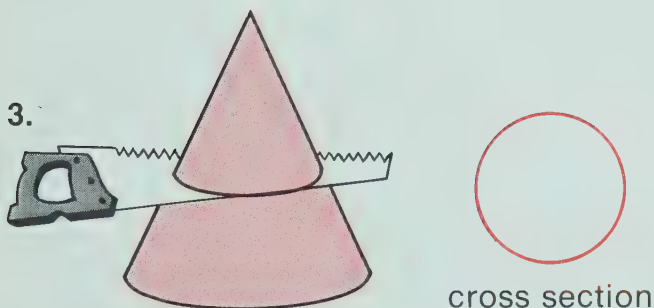
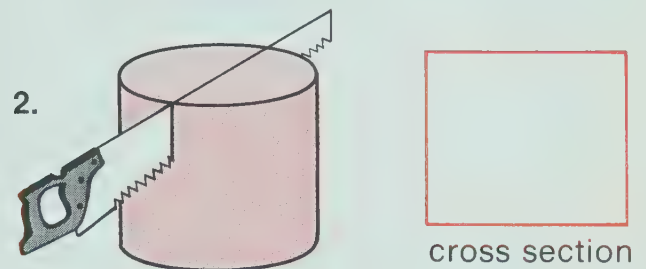
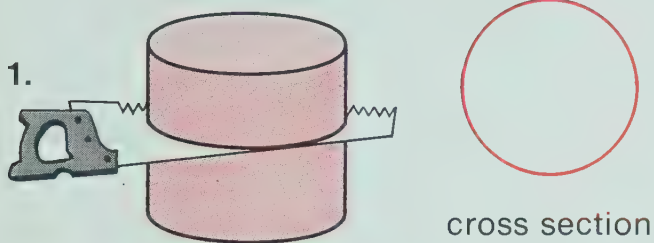
This page is extremely open-ended in terms of problem solving. Try to encourage children to put in realistic prices for the items.



The figure below shows a cross section. Think about cutting the figure.



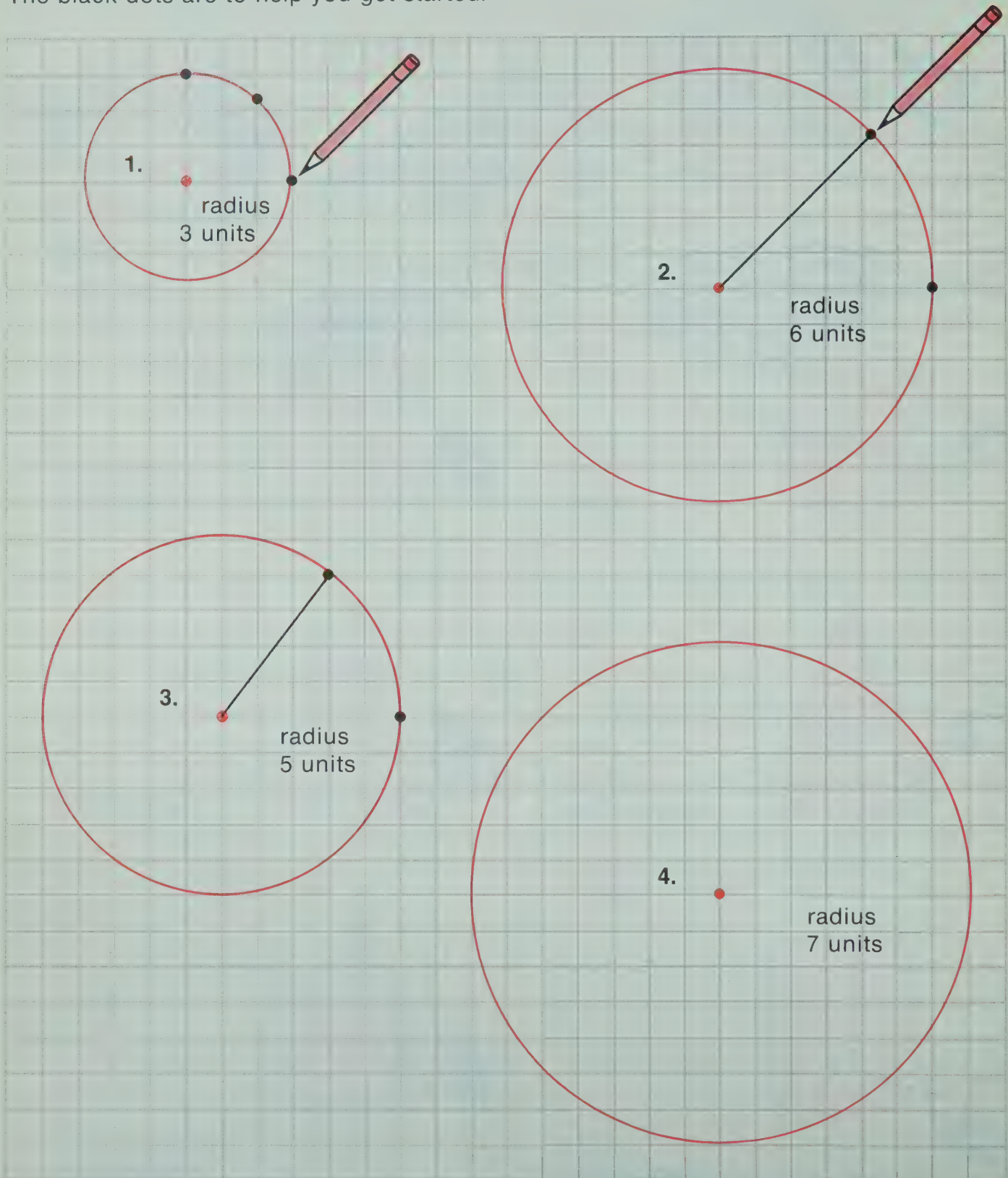
Draw a picture of the cross section for each cut shown below.  
Part 1 is an example.



One of the best ways for helping children understand this type of exercise is to actually have them do the cutting. This can best be accomplished by using figures made out of clay or other material that can be easily cut.

## ● Sketching Circles

When you **sketch** figures, you use **only** your pencil. Can you find a way to mark points on graph paper to help you **sketch** the circles called for below? The red dots are the centers. The black dots are to help you get started.



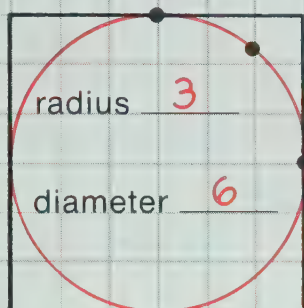
The children should be encouraged to count squares and parts of squares in order to estimate the location of various dots that will guide them in drawing or sketching the circle.



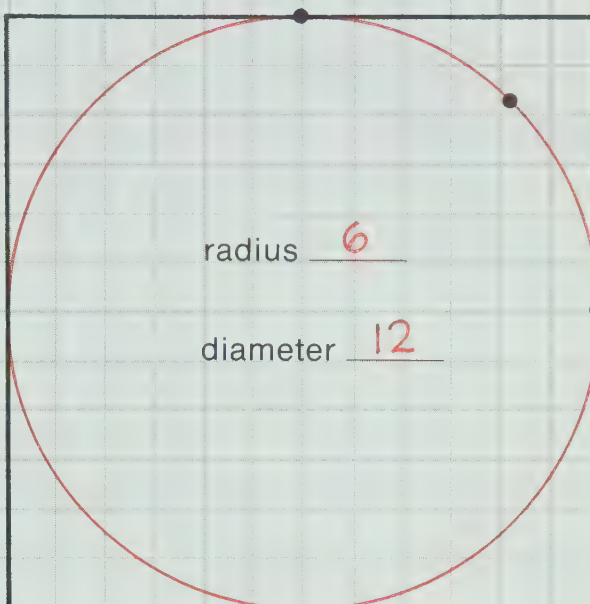
## ● Circles and Tangents

Sketch a circle inside each square so the sides of the square are tangent to the circle. Your work on page 56 should help you. Give the radius and diameter of each circle.

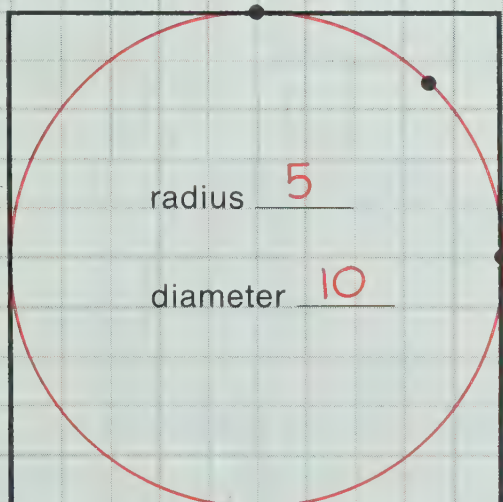
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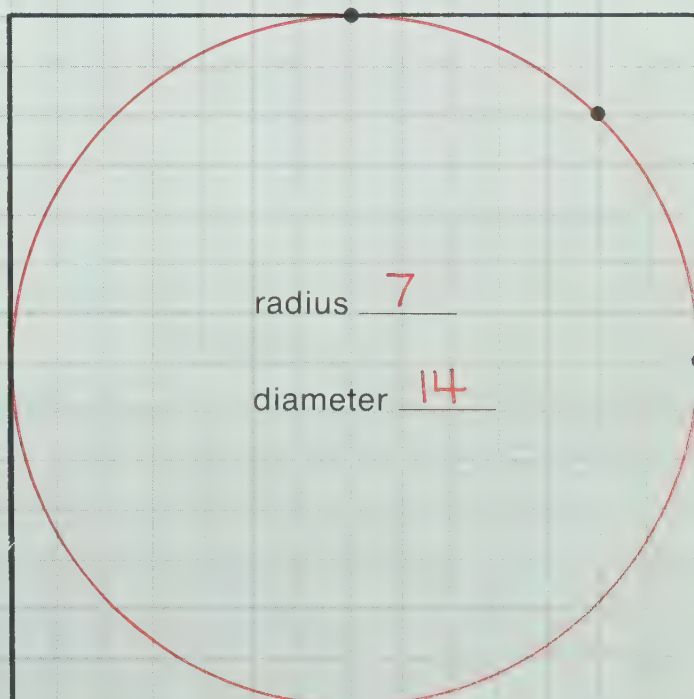
2.



3.



4.



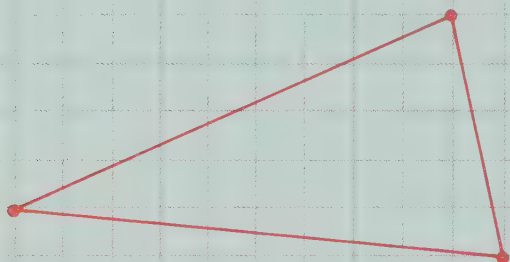
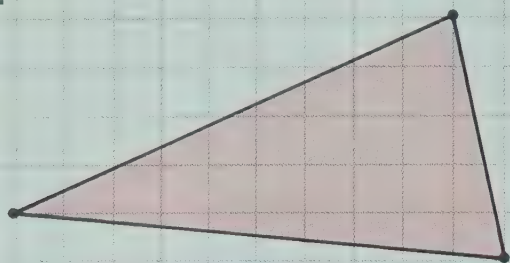
This lesson constitutes a follow-up lesson to the work started on page 56. Here, dots must be found in order to draw or sketch a circle that is inscribed in the squares.



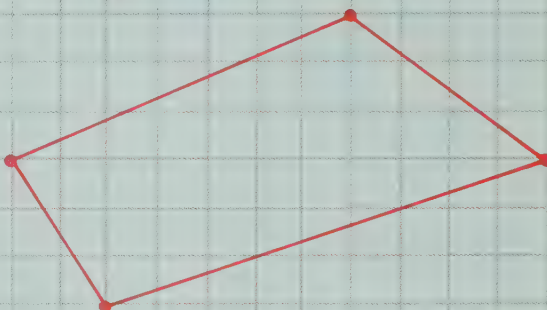
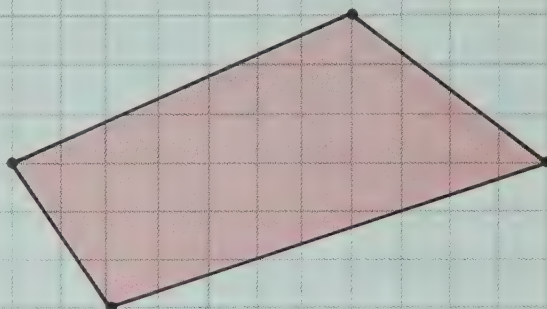
## Drawing Congruent Figures

Draw a figure so that it is congruent to the shaded figure.

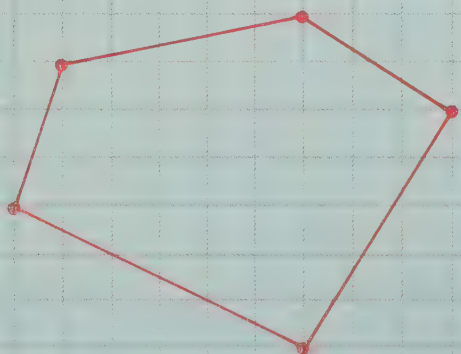
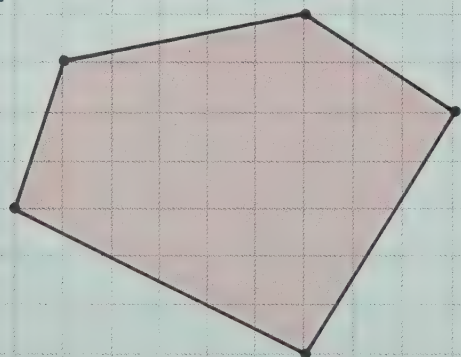
1.



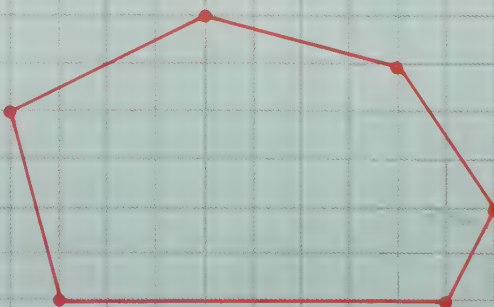
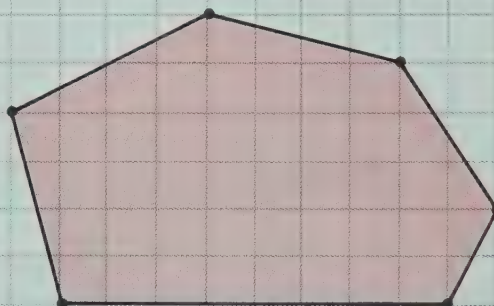
2.



3.



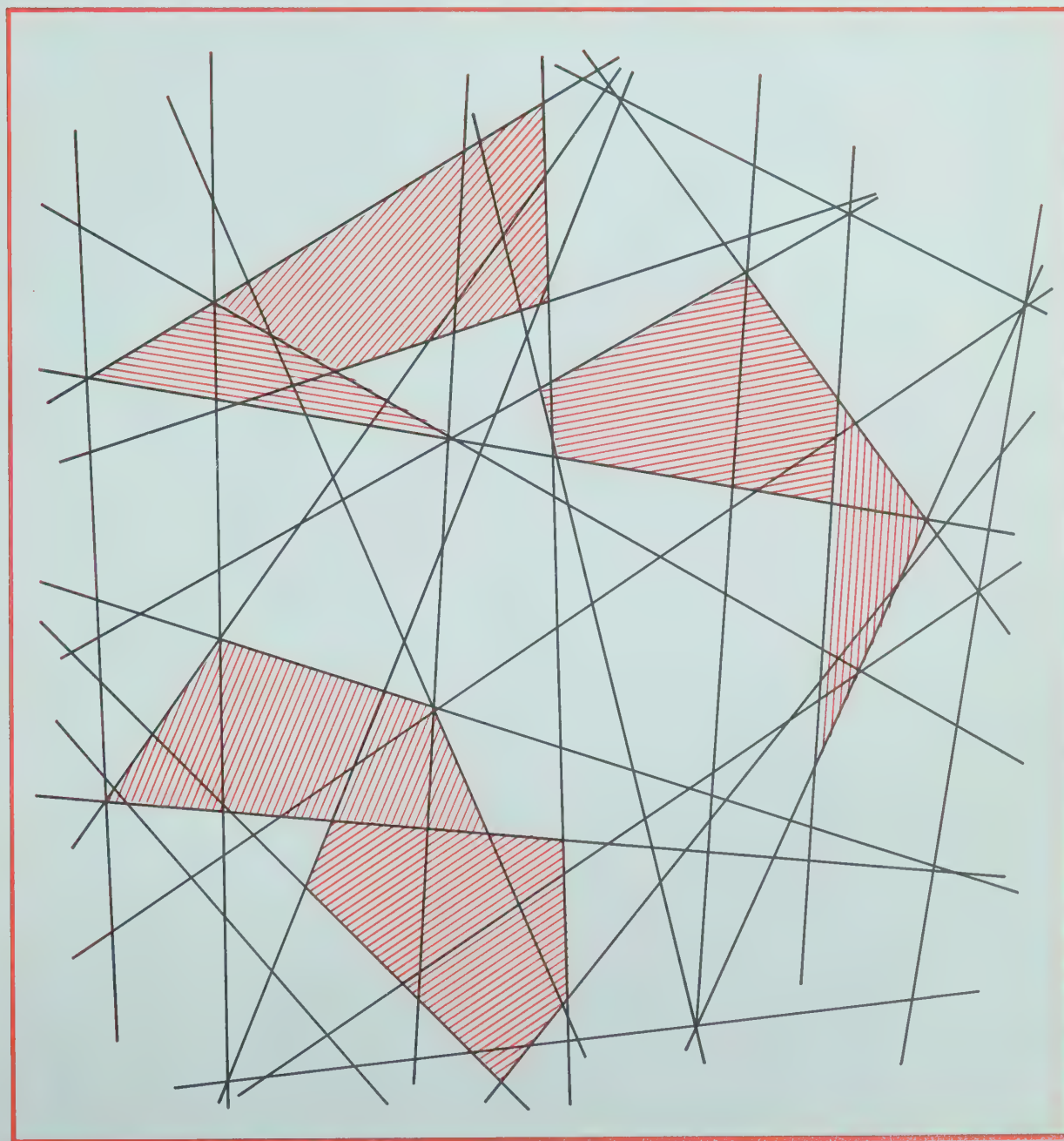
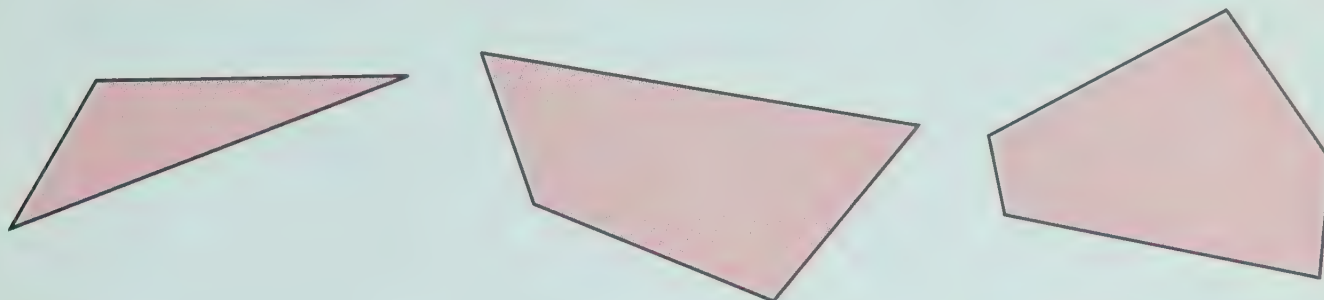
4.



The children should be encouraged to count squares on the graph paper so that the vertices of the polygon they draw are located in the same relative position as the one that is given.

## ● Congruent Figures

There are two figures “hidden” in the box below that are each congruent to the colored figures. How many can you find and color?

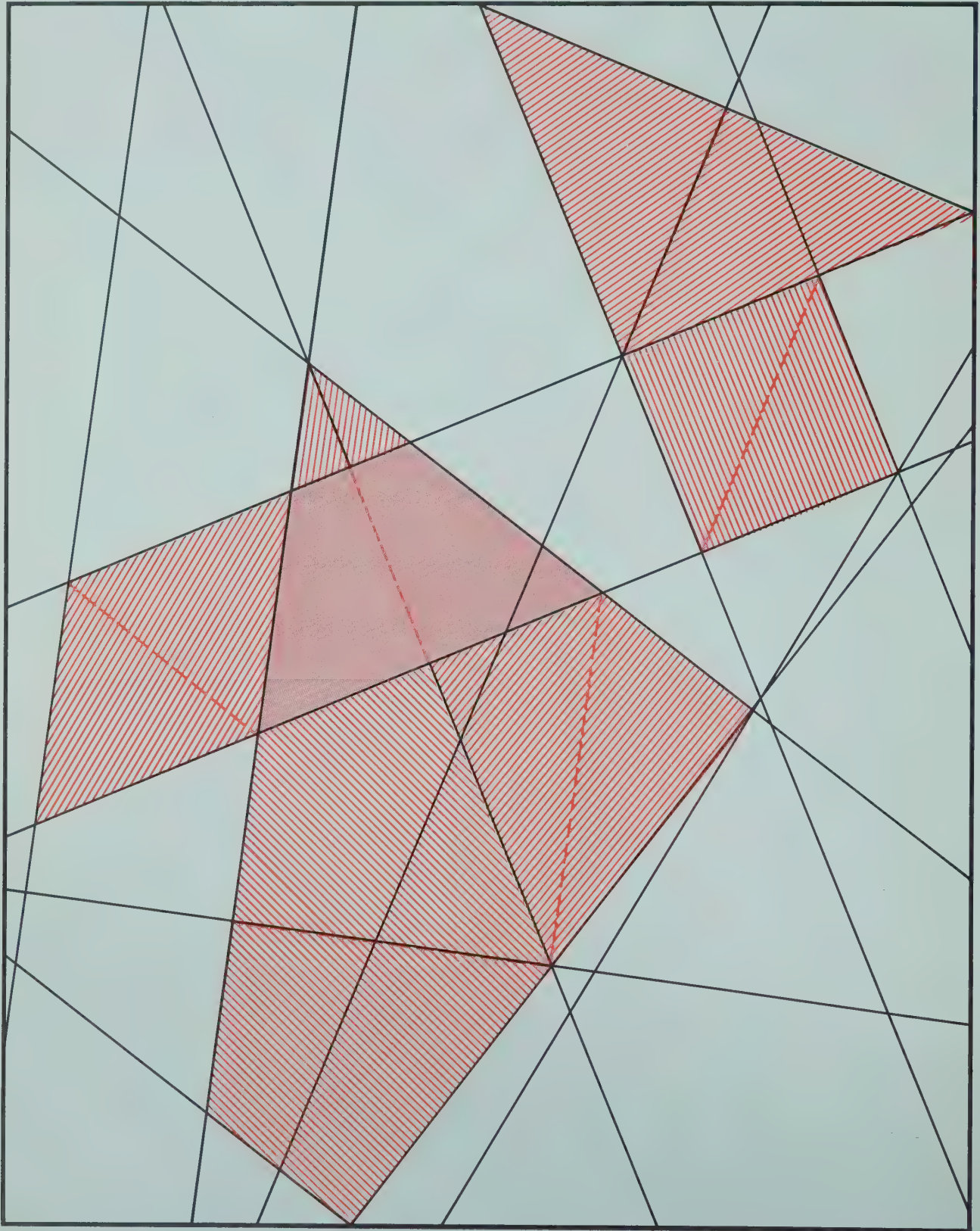


The most effective way to find the figure in the red box is to make a tracing of the given figure and slide it around until it coincides with one of the figures in the red box.



## ● Symmetrical Figures

There are at least 5 symmetrical figures hidden below. See how many you can find. Color them and draw a line of symmetry. One is shown as an example.



While the answers given suggest the intended symmetrical figures, the children may find others that are either symmetrical or very nearly symmetrical. You should accept any reasonable answer.



For each divisor make up as many problems as you can that have different remainders. You may not be able to complete all rows.

Sample answers:

1. Dividing by 4

$$\begin{array}{r} 3 \\ 4 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

All different → 0

$$\begin{array}{r} 3 \\ 4 \overline{) 13} \\ \underline{12} \\ 1 \end{array}$$

$$\begin{array}{r} 3 \\ 4 \overline{) 14} \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 3 \\ 4 \overline{) 15} \\ \underline{12} \\ 3 \end{array}$$

$$\begin{array}{r} 3 \\ 4 \overline{) \phantom{1}6} \\ \underline{\phantom{1}4} \\ 2 \end{array}$$

$$\begin{array}{r} 3 \\ 4 \overline{) \phantom{1}7} \\ \underline{\phantom{1}4} \\ 3 \end{array}$$

2. Dividing by 5

$$\begin{array}{r} 2 \\ 5 \overline{) 10} \\ \underline{10} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 5 \overline{) 11} \\ \underline{10} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \\ 5 \overline{) 12} \\ \underline{10} \\ 2 \end{array}$$

$$\begin{array}{r} 2 \\ 5 \overline{) 13} \\ \underline{10} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \\ 5 \overline{) 14} \\ \underline{10} \\ 4 \end{array}$$

$$\begin{array}{r} 2 \\ 5 \overline{) \phantom{1}5} \\ \underline{\phantom{1}5} \\ 0 \end{array}$$

3. Dividing by 6

$$\begin{array}{r} 2 \\ 6 \overline{) 12} \\ \underline{12} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 13} \\ \underline{12} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 14} \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 15} \\ \underline{12} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 16} \\ \underline{12} \\ 4 \end{array}$$

$$\begin{array}{r} 2 \\ 6 \overline{) 17} \\ \underline{12} \\ 5 \end{array}$$

4. Dividing by 8

$$\begin{array}{r} 2 \\ 8 \overline{) 16} \\ \underline{16} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 17} \\ \underline{16} \\ 1 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 18} \\ \underline{16} \\ 2 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 19} \\ \underline{16} \\ 3 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 20} \\ \underline{16} \\ 4 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 21} \\ \underline{16} \\ 5 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 22} \\ \underline{16} \\ 6 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) 23} \\ \underline{16} \\ 7 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) \phantom{2}4} \\ \underline{\phantom{2}0} \\ 4 \end{array}$$

$$\begin{array}{r} 2 \\ 8 \overline{) \phantom{2}5} \\ \underline{\phantom{2}4} \\ 1 \end{array}$$

Observe that when dividing by 4, there are four possible remainders; by 5, five remainders; 6, six remainders; 8, eight possible remainders. Therefore, not all of the spaces for dividing can be completed.

## ● Estimating Quotients

1. Study each problem. First write your estimate of the quotient in the red box. Then find the correct quotient. *Estimates will vary*

Estimate  
  
**A** 53R3  
 $4 \overline{) 215}$

Estimate  
  
**B** 65  
 $5 \overline{) 325}$

Estimate  
  
**C** 46R2  
 $8 \overline{) 370}$

Estimate  
  
**D** 70R4  
 $6 \overline{) 424}$

Estimate  
  
**E** 26R3  
 $7 \overline{) 185}$

Estimate  
  
**F** 54R2  
 $3 \overline{) 164}$

Estimate  
  
**G** 65R1  
 $5 \overline{) 326}$

Estimate  
  
**H** 29R2  
 $6 \overline{) 176}$

Estimate  
  
**I** 75R3  
 $7 \overline{) 528}$

Estimate  
  
**J** 59R4  
 $8 \overline{) 476}$

Estimate  
  
**K** 58R2  
 $4 \overline{) 234}$

Estimate  
  
**L** 41R6  
 $9 \overline{) 375}$

2. Give the remainder for each problem, without finding the quotient.

**A**  $2 \overline{) 376}$   
 R = 0

**B**  $2 \overline{) 587}$   
 R = 1

**C**  $5 \overline{) 287}$   
 R = 2

**D**  $5 \overline{) 615}$   
 R = 0

**E**  $5 \overline{) 964}$   
 R = 4

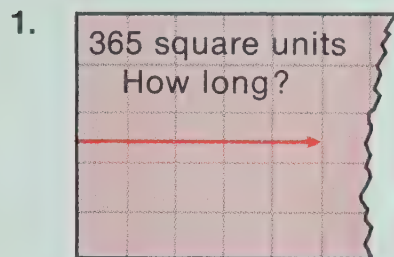
**F**  $5 \overline{) 339}$   
 R = 4

Exercise 1 is designed to help children improve their ability to estimate quotients.

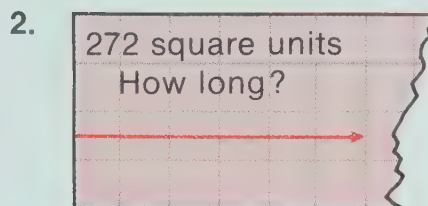
## ● Using Estimation In Division

First estimate how long the rectangle would be.  
Then use division to find out.

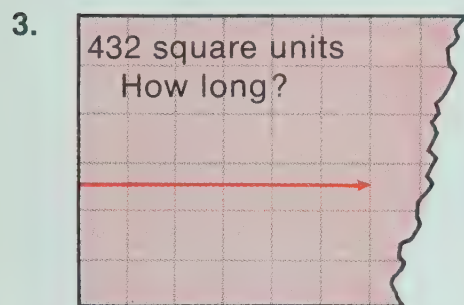
Estimates will vary



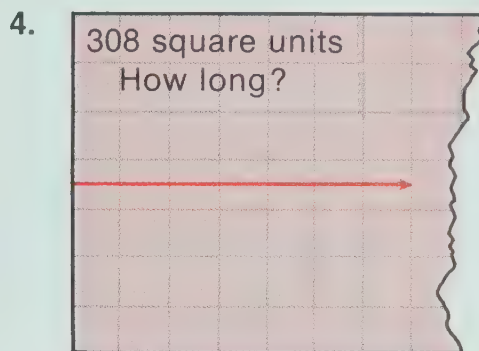
Estimate \_\_\_\_\_ Actual 73



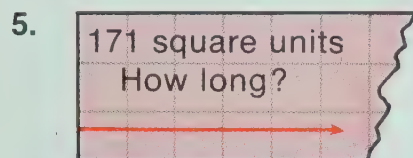
Estimate \_\_\_\_\_ Actual 68



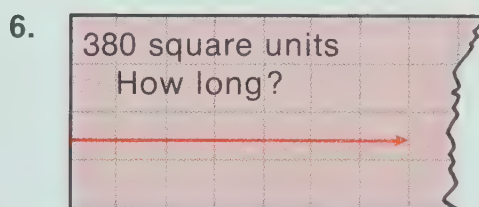
Estimate \_\_\_\_\_ Actual 72



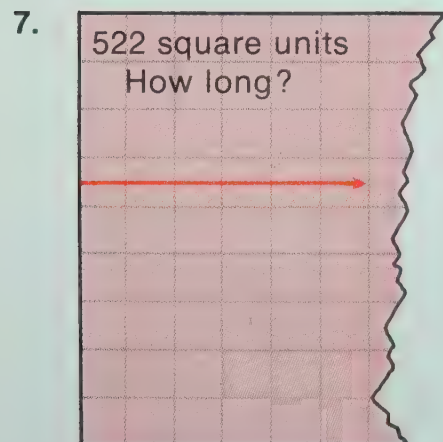
Estimate \_\_\_\_\_ Actual 44



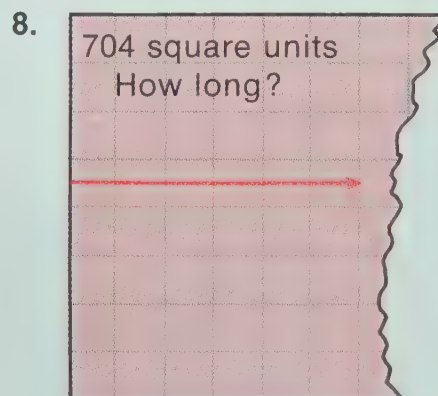
Estimate \_\_\_\_\_ Actual 57



Estimate \_\_\_\_\_ Actual 95



Estimate \_\_\_\_\_ Actual 88



Estimate \_\_\_\_\_ Actual 88

Children may need some guidance to observe that they can find the length of the rectangle by dividing the number of square units by its width. For example, in exercise 1, 365 should be divided by 5 to find the length.



## Finding Missing Digits

Find the missing digits. Hint: First find the dividend.

1.

$$\begin{array}{r}
 \phantom{0}3\phantom{0}4 \\
 6 \overline{) 208} \\
 \underline{180} \phantom{0} \\
 28 \phantom{0} \\
 \underline{24} \phantom{0} \\
 4
 \end{array}$$

2.

$$\begin{array}{r}
 \phantom{0}4\phantom{0}6 \\
 4 \overline{) 187} \\
 \underline{160} \phantom{0} \\
 27 \phantom{0} \\
 \underline{24} \phantom{0} \\
 3
 \end{array}$$

3.

$$\begin{array}{r}
 \phantom{0}5\phantom{0}2 \\
 7 \overline{) 366} \\
 \underline{350} \phantom{0} \\
 16 \phantom{0} \\
 \underline{14} \phantom{0} \\
 2
 \end{array}$$

4.

$$\begin{array}{r}
 \phantom{0}2\phantom{0}8 \\
 3 \overline{) 85} \\
 \underline{60} \phantom{0} \\
 25 \phantom{0} \\
 \underline{24} \phantom{0} \\
 1
 \end{array}$$

5.

$$\begin{array}{r}
 \phantom{0}2\phantom{0}4 \\
 9 \overline{) 216} \\
 \underline{180} \phantom{0} \\
 36 \phantom{0} \\
 \underline{36} \phantom{0} \\
 0
 \end{array}$$

6.

$$\begin{array}{r}
 \phantom{0}7\phantom{0}5 \\
 5 \overline{) 379} \\
 \underline{350} \phantom{0} \\
 29 \phantom{0} \\
 \underline{25} \phantom{0} \\
 4
 \end{array}$$

7.

$$\begin{array}{r}
 \phantom{0}5\phantom{0}1 \\
 6 \overline{) 309} \\
 \underline{300} \phantom{0} \\
 9 \phantom{0} \\
 \underline{6} \phantom{0} \\
 3
 \end{array}$$

8.

$$\begin{array}{r}
 \phantom{0}1\phantom{0}5 \\
 8 \overline{) 127} \\
 \underline{80} \phantom{0} \\
 47 \phantom{0} \\
 \underline{40} \phantom{0} \\
 7
 \end{array}$$

9.

$$\begin{array}{r}
 \phantom{0}2\phantom{0}1 \\
 7 \overline{) 149} \\
 \underline{140} \phantom{0} \\
 9 \phantom{0} \\
 \underline{7} \phantom{0} \\
 2
 \end{array}$$

10.

$$\begin{array}{r}
 \phantom{0}1\phantom{0}2 \\
 8 \overline{) 103} \\
 \underline{80} \phantom{0} \\
 23 \phantom{0} \\
 \underline{16} \phantom{0} \\
 7
 \end{array}$$

11.

$$\begin{array}{r}
 \phantom{0}4\phantom{0}6 \\
 9 \overline{) 416} \\
 \underline{360} \phantom{0} \\
 56 \phantom{0} \\
 \underline{54} \phantom{0} \\
 2
 \end{array}$$

12.

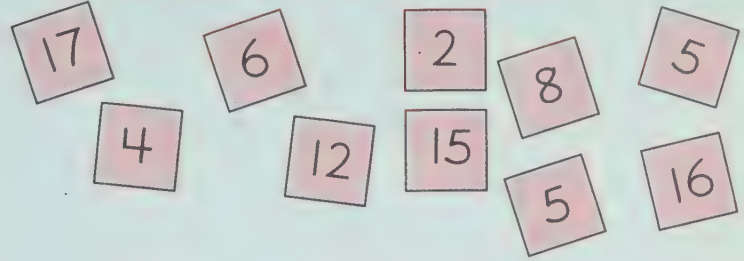
$$\begin{array}{r}
 \phantom{0}8\phantom{0}9 \\
 3 \overline{) 267} \\
 \underline{240} \phantom{0} \\
 27 \phantom{0} \\
 \underline{27} \phantom{0} \\
 0
 \end{array}$$

This lesson develops childrens' understanding of the idea that the dividend equals the divisor times the quotient plus the remainder. Once the dividend is found, the division can be completed in the usual manner.

## ● Finding Averages

Use this set of number cards for the problems below. Write your answers on the blank cards.

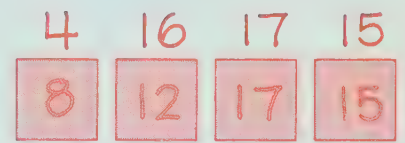
Possible answers shown



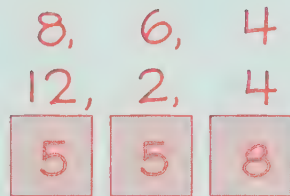
1. Find 3 cards so the average is 5.



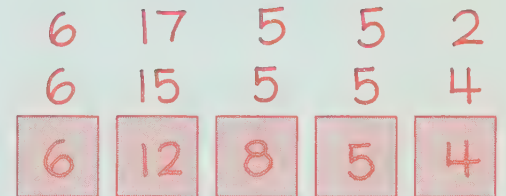
2. Find 4 cards so the average is 13.



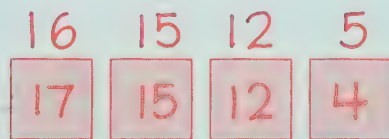
3. Find 3 cards so the average is 6.



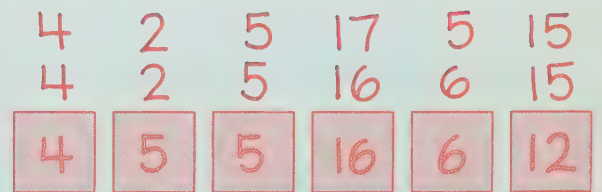
4. Find 5 cards so the average is 7.



5. Find 4 cards so the average is 12.



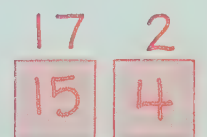
6. Find 6 cards so the average is 8.



7. Suppose you had these cards—  
6 8 2 What card could you choose so the average of all 4 would be 7.



8. Suppose you had these cards 12 8 6  
 What 2 cards could you choose so the average of all 5 would be 9.



Do not try to formulate rules for doing these unless the child discovers a "system." Some children may discover that the sum of the numbers on the cards must be the same as the product of the two numbers given in the problem. In exercise 1, that sum must be 15.

## ●3-Digit Quotients

First write your estimate of the quotient in the colored box. Then find the quotient. Next find the difference between your estimate and the quotient. Rate yourself as shown below. **Estimates will vary**

1.  $\begin{array}{r} 347 \\ 4 \overline{)1388} \end{array}$

Estimate

Difference

Score

2.  $\begin{array}{r} 854 \\ 3 \overline{)2562} \end{array}$

Estimate

Difference

Score

3.  $\begin{array}{r} 784 \\ 5 \overline{)3920} \end{array}$

Estimate

Difference

Score

4.  $\begin{array}{r} 265 \\ 6 \overline{)1590} \end{array}$

Estimate

Difference

Score

5.  $\begin{array}{r} 324 \\ 9 \overline{)2916} \end{array}$

Estimate

Difference

Score

6.  $\begin{array}{r} 648 \\ 7 \overline{)4536} \end{array}$

Estimate

Difference

Score

7.  $\begin{array}{r} 319 \\ 8 \overline{)2552} \end{array}$

Estimate

Difference

Score

8.  $\begin{array}{r} 917 \\ 6 \overline{)5502} \end{array}$

Estimate

Difference

Score

9.  $\begin{array}{r} 855 \\ 5 \overline{)4275} \end{array}$

Estimate

Difference

Score

Score each difference as follows:

- 10 or less—5 points
- Between 10 and 50—3 points
- 50 to 100—1 point
- over 100—0 points

Add your scores and rate yourself.

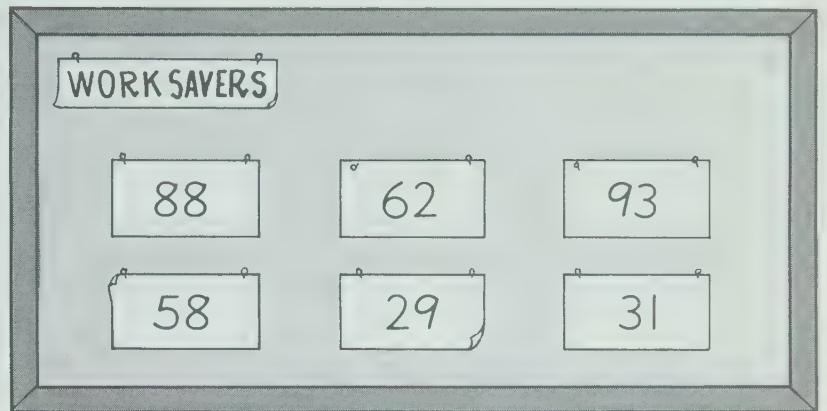
- 35-45      Fantastic
- 25-34      Excellent
- 15-24      Good
- 0-14        Keep trying

These exercises are designed to help children be as accurate as possible in making their estimates for quotients.



## ● Dividing by Multiples of 10

1. Find the quotients. Don't work any harder than you have to. All the answers are on the bulletin board. The remainder may or may not be zero.



A  $60 \overline{) 3500}$  58

B  $40 \overline{) 2497}$  62

C  $30 \overline{) 2640}$  88

D  $20 \overline{) 1770}$  88

E  $50 \overline{) 1551}$  31

F  $40 \overline{) 2320}$  58

G  $20 \overline{) 1864}$  93

H  $70 \overline{) 2035}$  29

I  $50 \overline{) 1491}$  29

J  $30 \overline{) 2800}$  93

K  $70 \overline{) 2187}$  31

L  $60 \overline{) 3737}$  62

2. Make up some problems of your own. The divisor should be a multiple of ten. The quotient is already given. answers will vary

A  $\overline{) \quad 58}$

B  $\overline{) \quad 39}$

C  $\overline{) \quad 84}$

D  $\overline{) \quad 75}$

E  $\overline{) \quad 63}$

F  $\overline{) \quad 71}$

G  $\overline{) \quad 28}$

H  $\overline{) \quad 96}$

By using logical reasoning concerning the numbers given on the work-saver board, the children should be able to arrive at the correct quotient without actually doing the dividing. For example, in problem 1A the child should be able to observe that the quotient is a little less than 60; since  $60 \times 60 = 3600$

## ● Missing Digit Problems

Find the missing digits.

1.  $\begin{array}{r} 77 \\ 5 \overline{)386} \\ 350 \\ \hline 36 \\ 35 \\ \hline 1 \end{array}$

2.  $\begin{array}{r} 91 \\ 8 \overline{)729} \\ 720 \\ \hline 9 \\ 8 \\ \hline 1 \end{array}$

3.  $\begin{array}{r} 993 \\ 6 \overline{)5963} \\ 5400 \\ \hline 563 \\ 540 \\ \hline 23 \\ 18 \\ \hline 5 \end{array}$

4.  $\begin{array}{r} 653 \\ 4 \overline{)2615} \\ 2400 \\ \hline 215 \\ 200 \\ \hline 15 \\ 12 \\ \hline 3 \end{array}$

5.  $\begin{array}{r} 356 \\ 7 \overline{)2492} \\ 2100 \\ \hline 392 \\ 350 \\ \hline 42 \\ 42 \\ \hline 0 \end{array}$

6.  $\begin{array}{r} 84 \\ 30 \overline{)2525} \\ 2400 \\ \hline 125 \\ 120 \\ \hline 5 \end{array}$

7.  $\begin{array}{r} 65 \\ 46 \overline{)3027} \\ 2760 \\ \hline 267 \\ 230 \\ \hline 37 \end{array}$

8.  $\begin{array}{r} 73 \\ 72 \overline{)5276} \\ 5040 \\ \hline 236 \\ 216 \\ \hline 20 \end{array}$

9.  $\begin{array}{r} 65 \\ 35 \overline{)2275} \\ 2100 \\ \hline 175 \\ 175 \\ \hline 0 \end{array}$

The chart below gives special names we will use on this page for special sets of numbers. Complete the chart. Then complete the tables and statements.

pink white gray

0	1	2
3	4	5
6	7	8
9	10	11
12	13	14
15	16	17
18	19	20
21	22	23
24	25	26
27	28	29
30	31	32
33	34	35
36	37	38
39	40	41
42	43	44
45	46	47
48	49	50
51	52	53
54	55	56
57	58	59
60	61	62

+	0	3	6	9
0	0	3	6	9
3	3	6	9	12
6	6	9	12	15
9	9	12	15	18

Table 1

×	0	3	6	9
0	0	0	0	0
3	0	9	18	27
6	0	18	36	54
9	0	27	54	81

Table 2

+	2	5	8	11
1	3	6	9	12
4	6	9	12	15
7	9	12	15	18
10	12	15	18	21

Table 3

×	1	4	7	10
0	0	0	0	0
3	3	12	21	30
6	6	24	42	60
9	9	36	63	90

Table 4

- In Table 1, the sum of a "pink" number and a "pink" number is a pink number.
- In Table 2, the product of a "pink" number and a "pink" number is a pink number.
- In Table 3, the sum of a "white" number and a "gray" number is a pink number.
- In Table 4, the product of a "pink" number and a "white" number is a pink number.

The table of numbers divided into three sets are based on 3. The pink numbers are multiples of three. The white numbers are each one more than a multiple of three, and the gray numbers are two more than a multiple of three.



## ● Factors and Products

All but two of the numbers below have at least one factor greater than 1 and less than the number itself. Give one or more such factors for each number that has them.

1. **38**

Factors 2, 19

2. **39**

Factors 3, 13

3. **37**

Factors \_\_\_\_\_

4. **51**

5. **85**

6. **91**

Factors 3, 17

Factors 5, 17

Factors 7, 13

7. **77**

8. **78**

9. **79**

Factors 7, 11

Factors 3, 26  
6, 13  
2, 39

Factors \_\_\_\_\_

10. **143**

11. **119**

12. **221**

Factors 11, 13

Factors 7, 17

Factors 13, 17

The intention on this page is that the child find at least one factor of the number. However, encourage them to find as many factors as they can.

## ● Greatest Common Factor

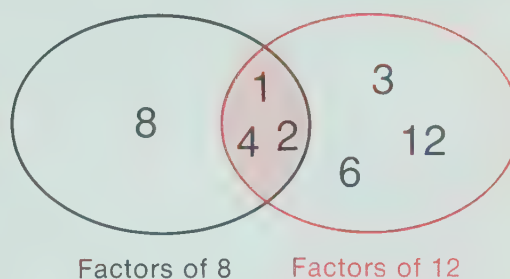
Study the example below of a **Venn Diagram**. Notice the shaded region is common to both sets. It contains the **common** factors of the two numbers.

The factors of 8: {1, 2, 4, 8}

The factors of 12: {1, 2, 3, 4, 6, 12}

List each set of factors. Then put them in the **Venn Diagram**. Give the greatest common factor (GCF) of the 2 numbers.

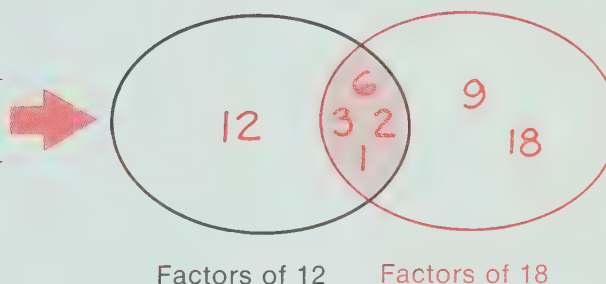
**Venn Diagram**



1. **A** Factors of 12: 1, 2, 3, 4, 6, 12

**B** Factors of 18: 1, 2, 3, 6, 9, 18

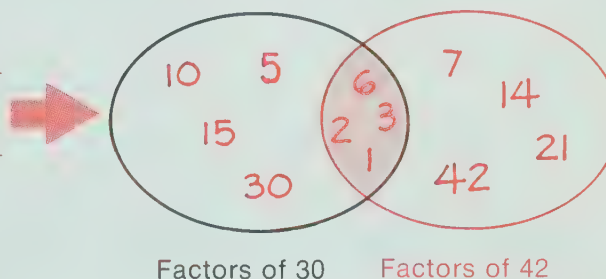
**C** The GCF of 12 and 18 is 6.



2. **A** Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

**B** Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

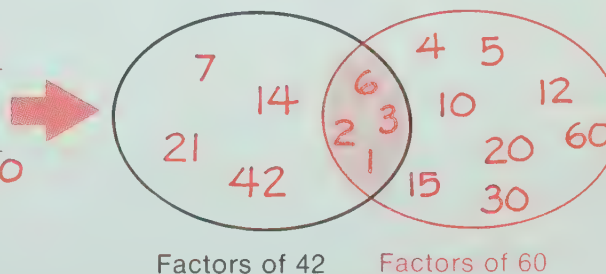
**C** The GCF of 30 and 42 is 6.



3. **A** Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

**B** Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

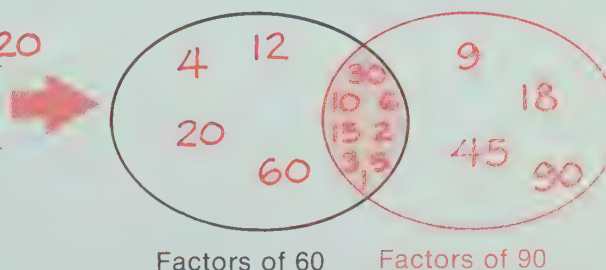
**C** The GCF of 42 and 60 is 6.



4. **A** Factors of 60: 1, 2, 3, 4, 5, 10, 12, 15, 20, 30, 60

**B** Factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

**C** The GCF of 60 and 90 is 30.



The main thing to help children understand with regard to Venn diagrams is that the numbers common to the two sets must go in the shaded region. That is, the region that is a part of both sets.

## ● Prime Numbers

1. Draw rectangles (or squares) for each of these areas. (No single rows or columns allowed.) Write the number of units inside the figure.

*Some answers will vary*

A 12 square units

B 10 square units

C 24 square units

D 40 square units

E 25 square units

F 27 square units

Example for A

12 square  
units

24  
square units

25  
square units

10  
square units

40  
square units

27 →  
square units

2. The prime numbers are not areas for rectangles like those above. Draw rectangles for these numbers that are not prime.

A 11

B 13

C 15

D 17

E 19

F 21

G 23

15  
square units

21  
square units

3. Could you use a rectangle to show whether or not 51 is prime?

3 × 17

This lesson emphasizes the fact that the non-prime numbers can be arranged into a square region or a rectangular array.



## ● Number Puzzlers

1. Try these puzzles.

**A** I'm a very odd teenager. In fact, I'm the only odd teenager

who isn't prime. Who am I? 15

**B** We're the oldest teenage primes. They call us twins because we differ

by just 2. Who are we? 17 and 19

**C** We're the only two primes that differ by just 1. We're side-

by-side. Who are we? 2 and 3

**D** We're the youngest pair of twins you'll ever find. Remember we're prime and differ by 2. Who are

we? 3 and 5

2. It is thought that every even number greater than 2 is the sum of two primes. How many pairs of primes can you find below? *answers will vary*

$$4 = \underline{2} + \underline{2}$$

$$28 = \underline{23} + \underline{5}$$

$$52 = \underline{47} + \underline{5}$$

$$6 = \underline{3} + \underline{3}$$

$$30 = \underline{23} + \underline{7}$$

$$54 = \underline{47} + \underline{7}$$

$$8 = \underline{3} + \underline{5}$$

$$32 = \underline{29} + \underline{3}$$

$$56 = \underline{53} + \underline{3}$$

$$10 = \underline{7} + \underline{3}$$

$$34 = \underline{31} + \underline{3}$$

$$58 = \underline{53} + \underline{5}$$

$$12 = \underline{7} + \underline{5}$$

$$36 = \underline{31} + \underline{5}$$

$$60 = \underline{53} + \underline{7}$$

$$14 = \underline{11} + \underline{3}$$

$$38 = \underline{31} + \underline{7}$$

$$62 = \underline{59} + \underline{3}$$

$$16 = \underline{13} + \underline{3}$$

$$40 = \underline{37} + \underline{3}$$

$$64 = \underline{59} + \underline{5}$$

$$18 = \underline{13} + \underline{5}$$

$$42 = \underline{37} + \underline{5}$$

$$66 = \underline{59} + \underline{7}$$

$$20 = \underline{17} + \underline{3}$$

$$44 = \underline{37} + \underline{7}$$

$$68 = \underline{61} + \underline{7}$$

$$22 = \underline{19} + \underline{3}$$

$$46 = \underline{43} + \underline{3}$$

$$70 = \underline{67} + \underline{3}$$

$$24 = \underline{19} + \underline{5}$$

$$48 = \underline{43} + \underline{5}$$

$$72 = \underline{67} + \underline{5}$$

$$26 = \underline{23} + \underline{3}$$

$$50 = \underline{43} + \underline{7}$$

$$74 = \underline{67} + \underline{7}$$

The answers to exercise 2 are not necessarily unique.

## ● A Number Theory Puzzle

### ACROSS

1. First 2-digit multiple of 7
3. Last prime before 41
5.  $10,000 + 1000 + 100 + 10 + 1$
9. Hundreds and tens places in 2035
10. Factors: 1, 2, 4, 8, 16
11. 6 sixes
12.  $10^2 - 10$
13. Smallest 2-digit number
14. One of it's 3 factors is 7
16. First prime  $< 60$
17. Between 29 and it's twin.
18. Smallest 2-digit prime.
20. 6, 7, 5, 7, 4, 7, —, —, —, —.
22. A product of primes,  $3 \times 3 \times 7 \times 7$

26. 2, two, II, 2, two
29. First 4 multiples of 3, reverse order.
30. A thousand more than 1135.
32. First 5 even numbers.
33.  $10^2 + 94$
34. 1000 more than 48 down.
37. Twin prime with 43.
39. Only prime in the nineties.
41. Greatest common factor of 30 and 40.
42. Smallest teen prime.
43. Twin prime with 11.
45. Prime factors: 7 and 13.
46. Last 2-digit multiple of 10.
47. Prime factors: 5 and 11.
49. Teen number whose only prime factor is 2.
50. Even digits, reverse order.
51. 0, 12, 24, 36, 48, 60, ?
52. Twin prime with 17.

### DOWN

1. First 3-digit prime.
2. It's prime factors are 23, 11, and 17.
3. First prime after 29.
4. Largest number with 3, 4, 6, and 7.
5. First 4 odd numbers.
6. One of it's 3 factors is 13.
7.  $(2 \times 10^2) - 7$
8. Smallest 3-digit number.
15. Largest 2-digit prime.
19. Smallest number with the digits 0, 1, 2, 3, 4.
21. Smallest prime in all 4 places.
22. forty-six thousand, eight hundred four.
23. Twin prime with 41.
24. Prime factors: 2, 2, 5, 5.
25. One of it's 3 factors is 11.
27. One of it's 3 factors is 5.
28. Prime factors: 2, 2, 2, 5, 5, 5.
29. 3 thirty-twos
31. Largest teen prime.
34. Smallest 4-digit number.
35. Prime factors: 3 and 17.
36. First 4 primes.
38. 11 hundreds and 11.
39.  $1000 - \text{smallest prime}$ .
40. One minute past quarter after 7.
41.  $200 - (2 \times 2 \times 2)$
44. 0, —, —, —, 12, 15, 18, 21.
48.  $13 \times 2 \times 2$

1	2		3	4			5	6		7	8
1	4		3	7			1	1	1	1	1
9	0	3		1	6			3	6		9
13	1	0			4	9		5	9		3
	18	19		20	3	7	2	7			
		0					2			22	23
25		26	27	2	2	2	2		29	9	6
30	31	3	5			0	2	4	6	8	
33	1	9	4			0				0	
				34	1	0	35	36		37	38
39	40		41	1	0		42	1	3		43
45	9	1		46	9	0			47	48	49
50	8	6	4	2	0			51	7	2	52
											1
											9

There are 60 minutes in an hour. Color the fractional part of the clock (1 hour) indicated by the number of minutes. Then give the fraction to tell what part you shaded.

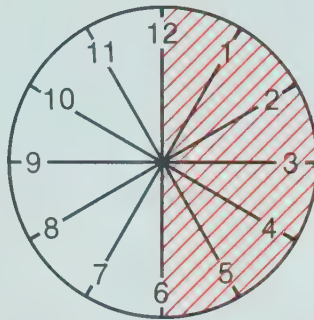
EXAMPLE:



15 minutes

Part of an hour.  $\frac{1}{4}$

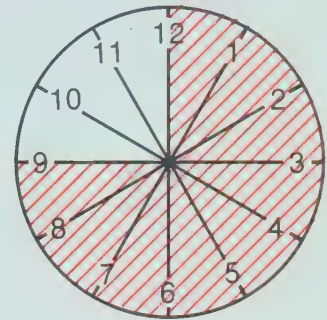
1.



30 minutes

Part of an hour.  $\frac{1}{2}$

2.



45 minutes

Part of an hour.  $\frac{3}{4}$

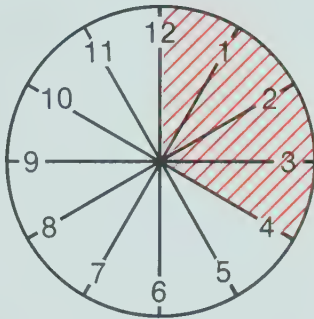
3.



10 minutes

Part of an hour.  $\frac{1}{6}$

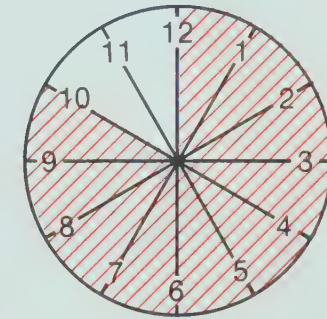
4.



20 minutes

Part of an hour.  $\frac{2}{3}$

5.



50 minutes

Part of an hour.  $\frac{5}{6}$

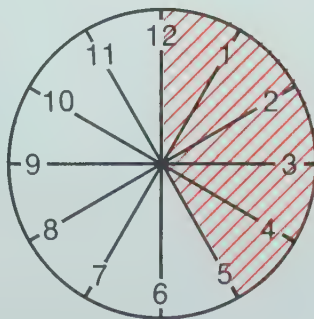
6.



5 minutes

Part of an hour.  $\frac{1}{12}$

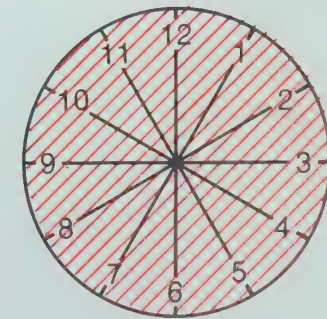
7.



25 minutes

Part of an hour.  $\frac{5}{12}$

8.



60 minutes

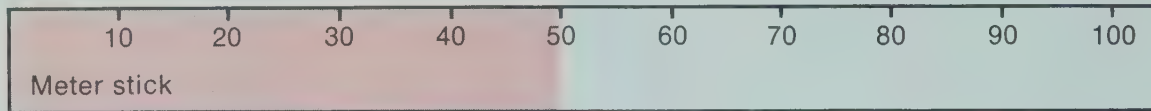
Part of an hour.  $\frac{1}{1}$



## Fractional Parts of a Meter

There are 100 centimeters in a meter. Think of the rulers below as meter sticks reduced in size. Shade the given fractional part. Then answer the questions.

EXAMPLE:

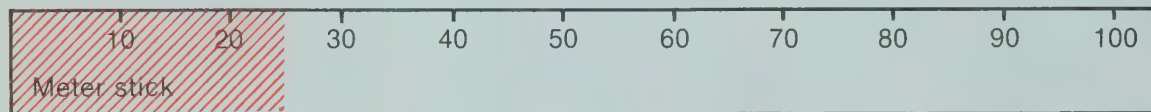


Shade  $\frac{1}{2}$

How many centimeters did you shade? 50

What fractional part (in hundredths) did you shade?  $\frac{50}{100}$

1.



Shade  $\frac{1}{4}$

How many centimeters did you shade? 25

What fractional part (in hundredths) did you shade?  $\frac{25}{100}$

2.

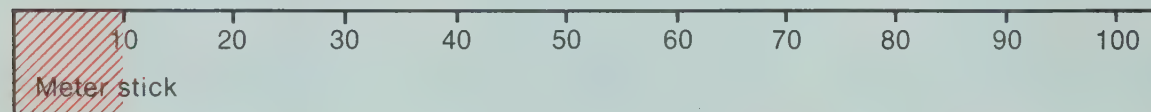


Shade  $\frac{3}{4}$

How many centimeters did you shade? 75

What fractional part (in hundredths) did you shade?  $\frac{75}{100}$

3.

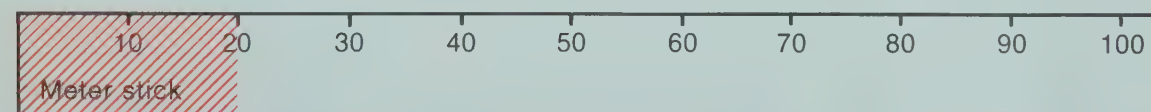


Shade  $\frac{1}{10}$

How many centimeters did you shade? 10

What fractional part (in hundredths) did you shade?  $\frac{10}{100}$

4.

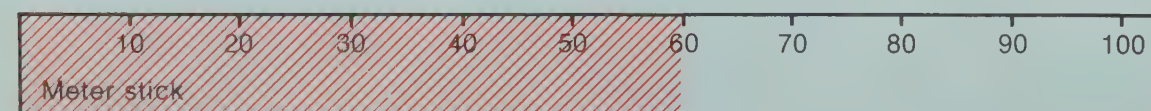


Shade  $\frac{1}{5}$

How many centimeters did you shade? 20

What fractional part (in hundredths) did you shade?  $\frac{20}{100}$

5.



Shade  $\frac{3}{5}$

How many centimeters did you shade? 60

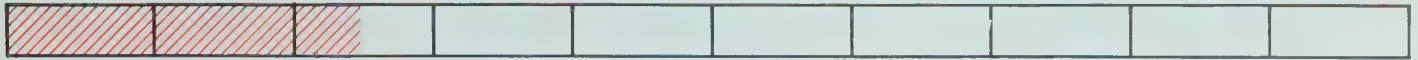
What fractional part (in hundredths) did you shade?  $\frac{60}{100}$

Don't attempt to teach children how to find the amount of the meter stick to shade. Rather, let them attempt to find their own method. Emphasize the fact that once the meter stick is shaded, it's easy to find the number of hundredths involved.

## ● Finding Missing Fractions

Give the missing fractions. Then color a fractional part of the rod for the set of equivalent fractions. The rod is divided into 10ths to help you color.

1.  $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}, \frac{8}{32}, \frac{9}{36}, \frac{10}{40}, \dots$



2.  $\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}, \frac{21}{35}, \frac{24}{40}, \frac{27}{45}, \frac{30}{50}, \dots$



3.  $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}, \dots$



4.  $\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}, \frac{30}{48}, \frac{35}{56}, \frac{40}{64}, \frac{45}{72}, \frac{50}{80}, \dots$



5.  $\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \frac{42}{60}, \frac{49}{70}, \frac{56}{80}, \frac{63}{90}, \frac{70}{100}, \dots$



6.  $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}, \frac{12}{18}, \frac{14}{21}, \frac{16}{24}, \frac{18}{27}, \frac{20}{30}, \dots$



7.  $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}, \frac{8}{16}, \frac{9}{18}, \frac{10}{20}, \dots$



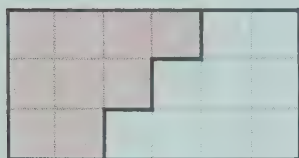
By necessity the children will need to estimate the amount of the rod to color. Once colored, the children will automatically have an estimate of that fractional number in tenths, since each of the rods is divided into tenths.



## Tenths and Hundredths

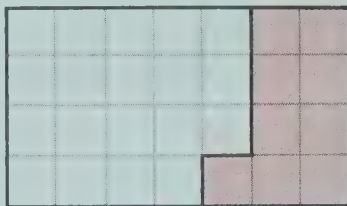
1. Much of your work with fractions will use tenths and hundredths.  
Estimate the numerator of each fraction.

A



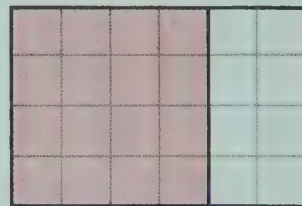
About  $\frac{5}{10}$  is shaded.

B



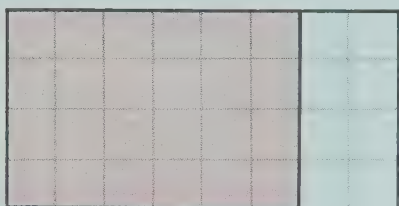
About  $\frac{3}{10}$  is shaded.

C



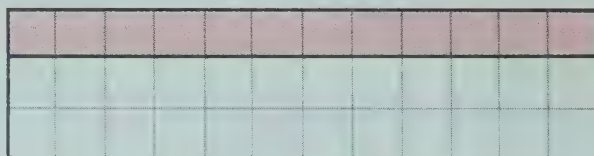
About  $\frac{7}{10}$  is shaded.

D



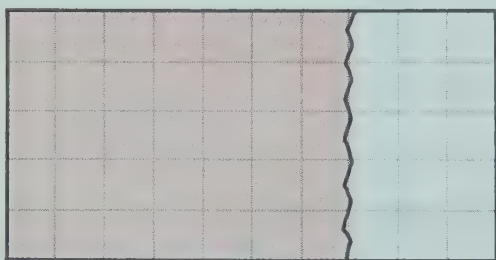
About  $\frac{75}{100}$  is shaded.

E



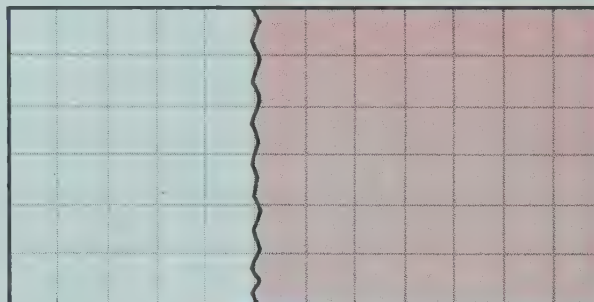
About  $\frac{33}{100}$  is shaded.

F



About  $\frac{70}{100}$  is shaded.

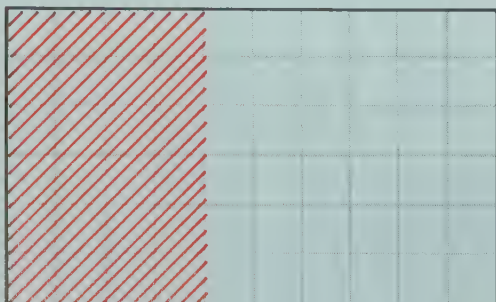
G



About  $\frac{56}{100}$  is shaded.

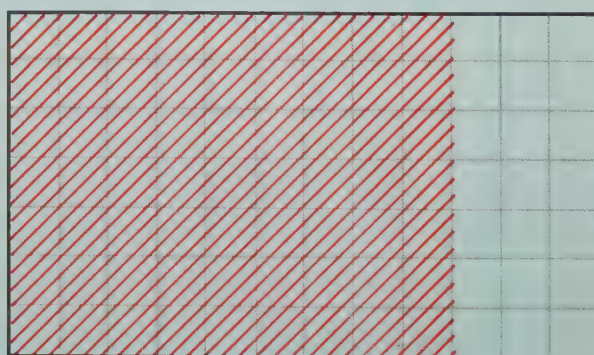
2. Shade the given amount as close as you can.

A



Shade about  $\frac{4}{10}$ .

B



Shade about  $\frac{75}{100}$ .

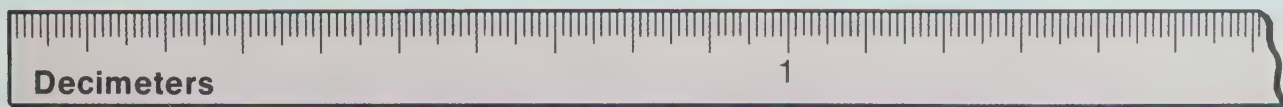
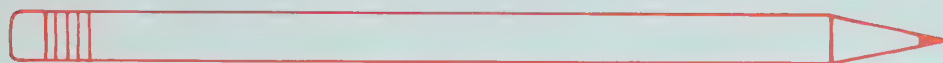
Allow considerable leeway in the estimates that children make for the number of tenths and hundredths shaded in these figures.



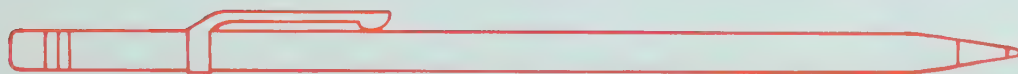
## ● Using Improper Fractions

The unit for the ruler below is the decimeter (10 centimeters). See if you can draw the figures suggested.

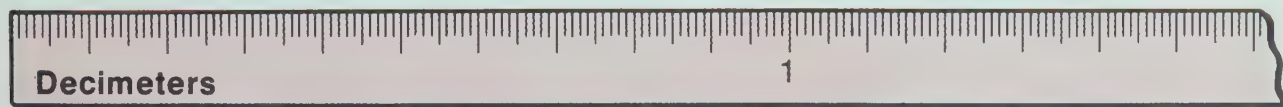
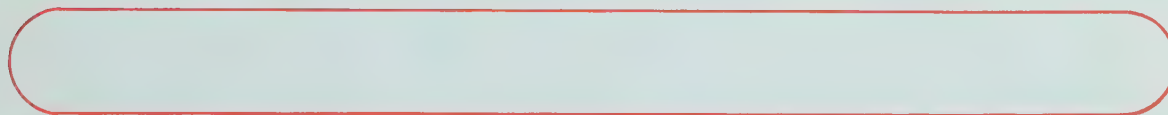
1. Draw a pencil that is  $\frac{12}{10}$  decimeters long.



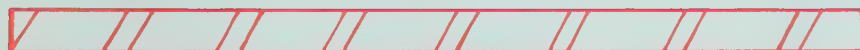
2. Draw a ball point pen that is  $\frac{13}{10}$  decimeters long.



3. Draw an ice cream stick that is  $\frac{3}{2}$  decimeters long.



4. Draw a drinking straw that is  $\frac{110}{100}$  decimeters long.



5. Draw a tooth brush that is  $\frac{6}{5}$  decimeters long.



It will be helpful if children understand that each centimeter is  $\frac{1}{10}$  of a decimeter.

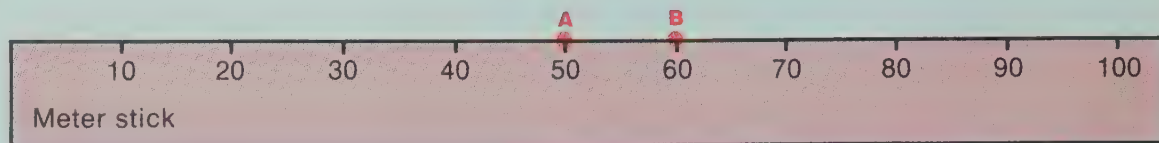
## ● Exploring Fractions that are not Equivalent

Can you find a way to mark points on the meter stick for each length.  
After you do, ring the greater length.

EXAMPLE:

A  $\frac{1}{2}$  meter

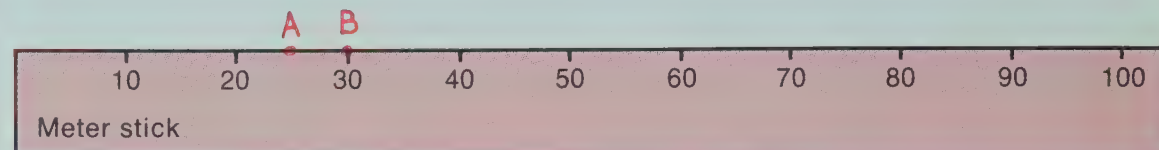
B  $\frac{6}{10}$  meter



1.

A  $\frac{1}{4}$  meter

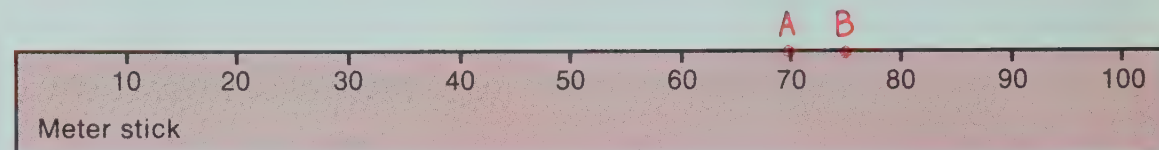
B  $\frac{3}{10}$  meter



2.

A  $\frac{7}{10}$  meter

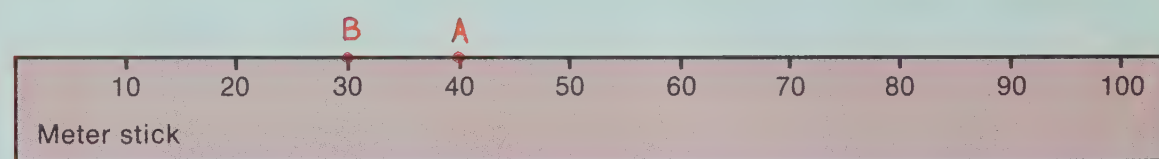
B  $\frac{3}{4}$  meter



3.

A  $\frac{2}{5}$  meter

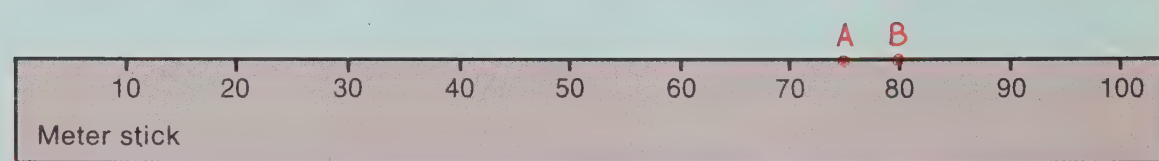
B  $\frac{3}{10}$  meter



4.

A  $\frac{3}{4}$  meter

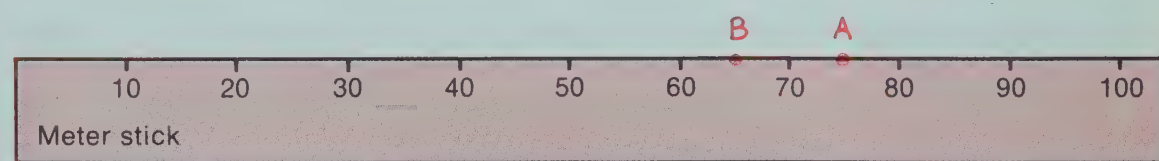
B  $\frac{8}{10}$  meter



5.

A  $\frac{3}{4}$  meter

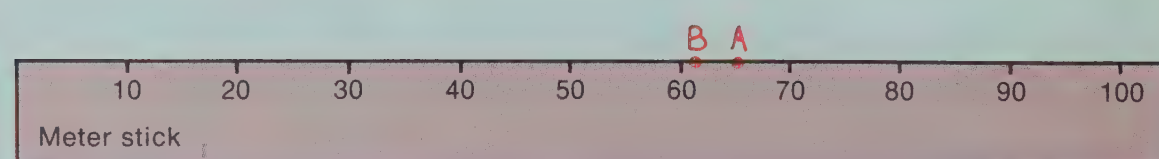
B  $\frac{2}{3}$  meter



6.

A  $\frac{2}{3}$  meter

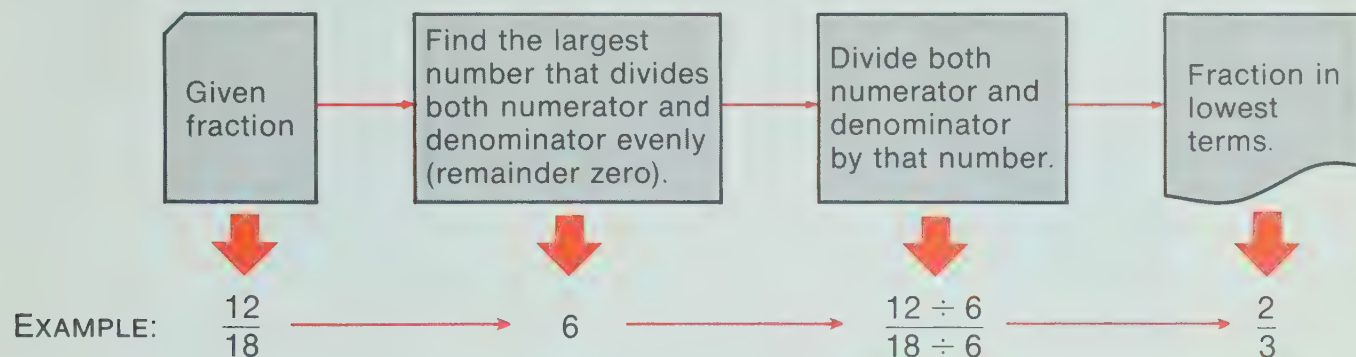
B  $\frac{5}{8}$  meter



Encourage children to create their own methods for finding the point on the meter stick for the given length.

## Finding Lowest-terms Fractions

1. The flow chart below shows how to find a lowest-terms fraction that is equivalent to the given fraction.



A Now you try one.  $\frac{10}{20} \rightarrow \square \rightarrow \frac{\square}{\square} \rightarrow \frac{\square}{\square}$

If you did the work correctly the lowest-terms fraction should be  $\frac{1}{2}$ .

B Try another.  $\frac{15}{30} \rightarrow \square \rightarrow \frac{\square}{\square} \rightarrow \frac{\square}{\square}$

This time you should get  $\frac{1}{2}$ .

2. Now find the lowest-terms fraction for each of these fractions. The answers are in the answer box.

A  $\frac{6}{8} = \frac{3}{4}$

B  $\frac{8}{12} = \frac{2}{3}$

C  $\frac{9}{18} = \frac{1}{2}$

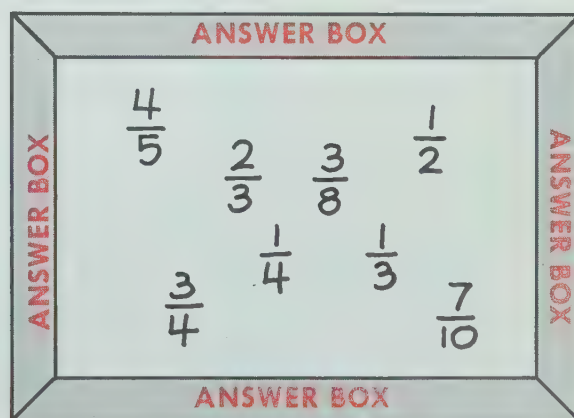
D  $\frac{21}{30} = \frac{7}{10}$

E  $\frac{12}{32} = \frac{3}{8}$

F  $\frac{5}{15} = \frac{1}{3}$

G  $\frac{6}{24} = \frac{1}{4}$

H  $\frac{24}{30} = \frac{4}{5}$



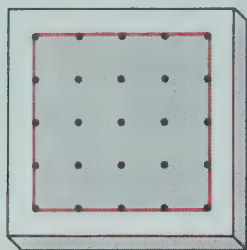
It will be helpful if you check the two exercises at the top of the page before having the children attempt exercise 2. The answer box is given only as a check for independent work.



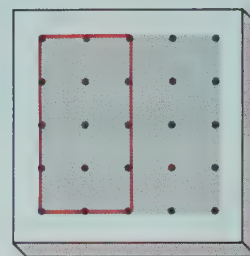
## Fractions and Area

1. Think of the figures below as boards with 25 nails driven in part way.  
Think of the colored outlines as rubber bands around the nails.

If this rubber band encloses an area of 1,



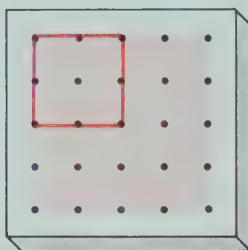
what area does this rubber band enclose?



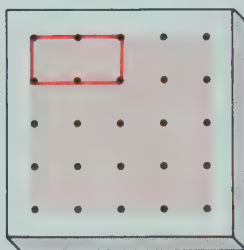
$$\frac{1}{2}$$

2. Think of the shaded part as having area 1. Show an area on the nail board for each fraction.

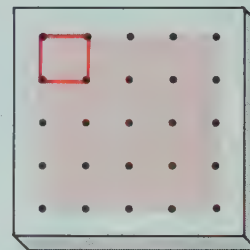
A  $\frac{1}{4}$



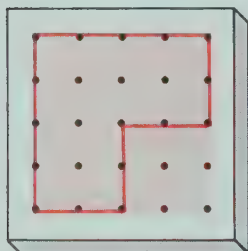
B  $\frac{1}{8}$



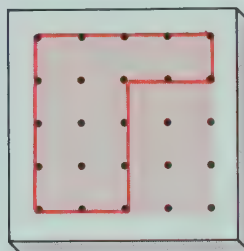
C  $\frac{1}{16}$



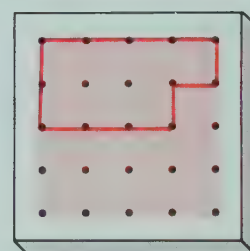
D  $\frac{3}{4}$



E  $\frac{5}{8}$



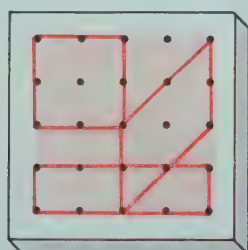
F  $\frac{7}{16}$



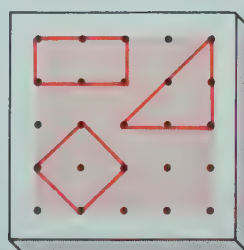
3. Show each fraction with as many different shaped regions as you can.  
You may want to use the "extra" nailboards to show your answers.

Sample answers

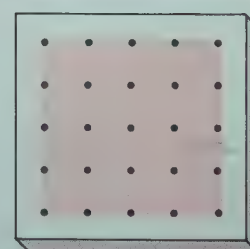
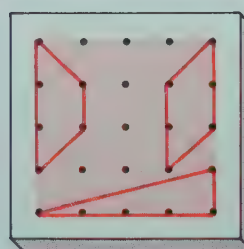
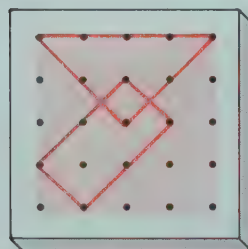
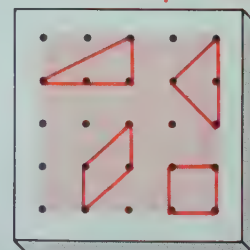
A  $\frac{1}{4}$



B  $\frac{1}{8}$

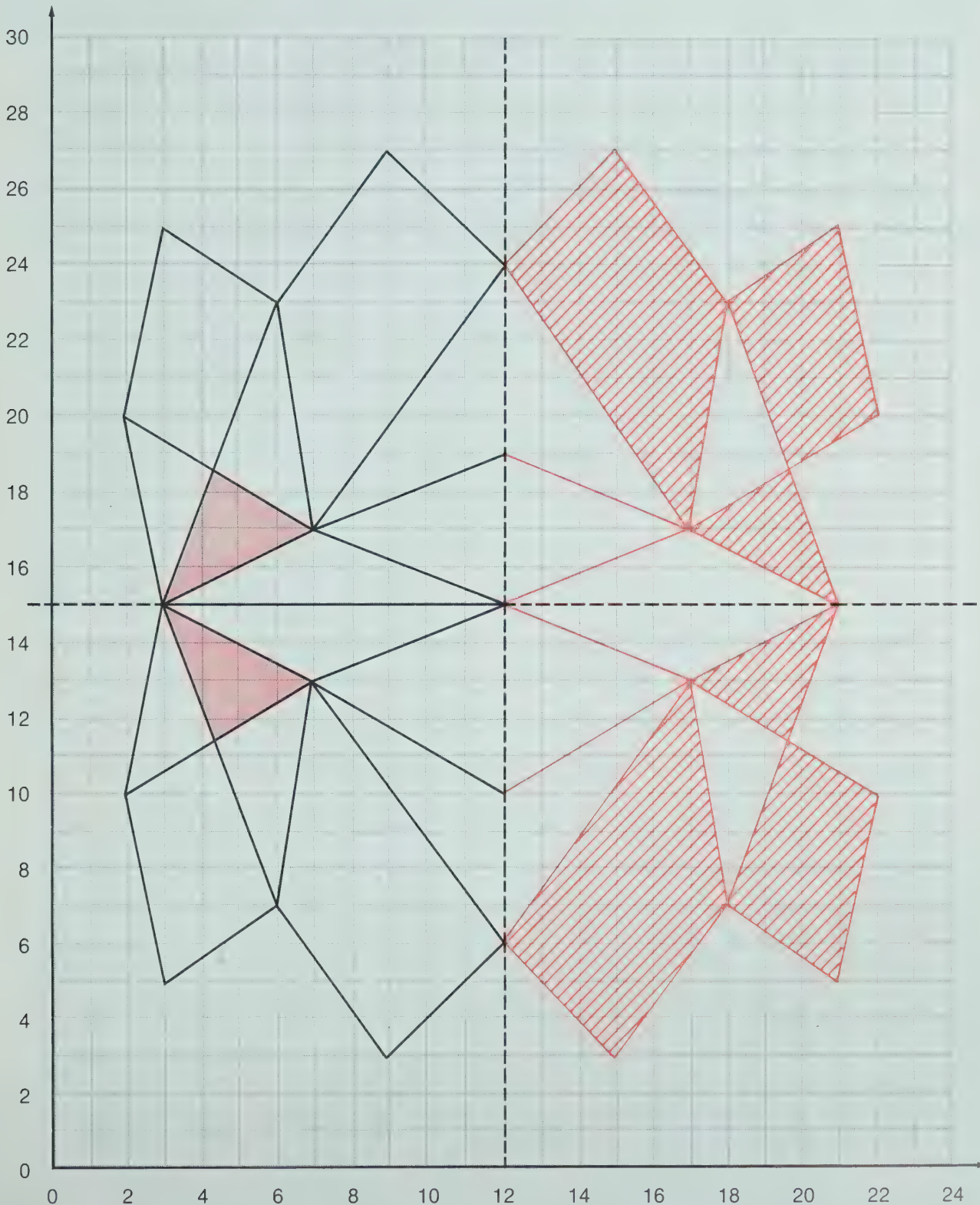


C  $\frac{1}{16}$



The most effective way to work with this page is for each child to have a geoboard. If this is not possible, it may be helpful if the children have extra dot paper to use in attempting to find these areas.

Use coordinates to help you draw the other half of this symmetrical design. Can you complete coloring the design so it can be folded along either dashed line and the colors will “match.”



Try making a symmetrical design of your own on another sheet of graph paper

While it is intended that children use coordinates to help them complete this figure, any method they devise is acceptable.



## ● Using Coordinates to Sketch Circles

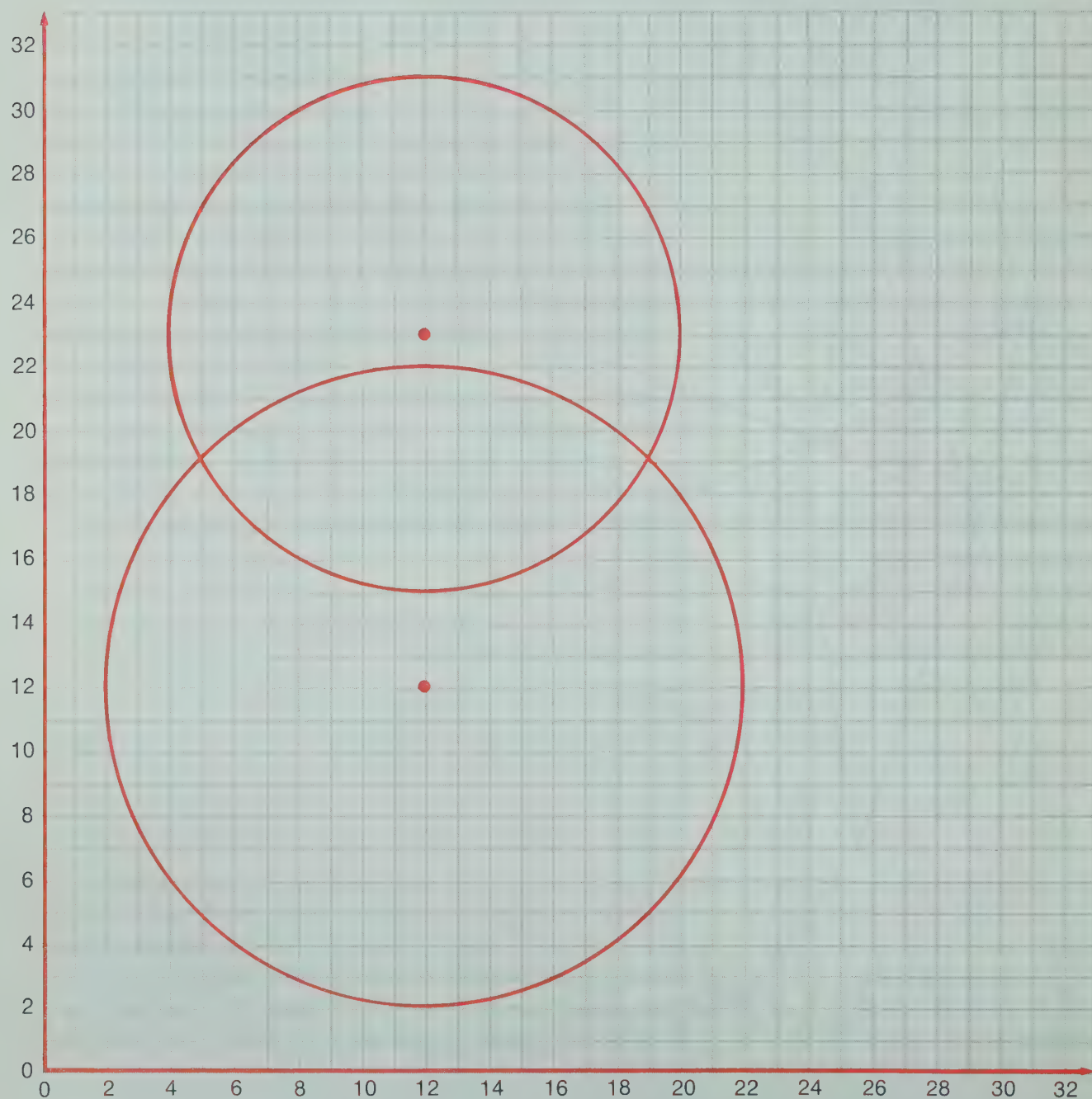
Use the “coordinate clues” below to help you sketch two circles.  
Remember, circles are symmetrical figures.

### First Circle

- Clue 1 Center: (12, 12)  
Clue 2 Radius: 10  
Clue 3 Passes through (18, 4) and (20, 6)

### Second Circle

- Clue 1 Center: (12, 23)  
Clue 2 Radius: 8  
Clue 3 Passes through (approximate):  
(5, 27) and (8, 30)



If you made your sketches carefully, the two circles should intersect at about (5, 19) and (19, 19).

In sketching these circles, the children can call upon the work they did earlier in sketching circles on graph paper. The clue at the bottom of the page is given as a check on the accuracy of their sketches.

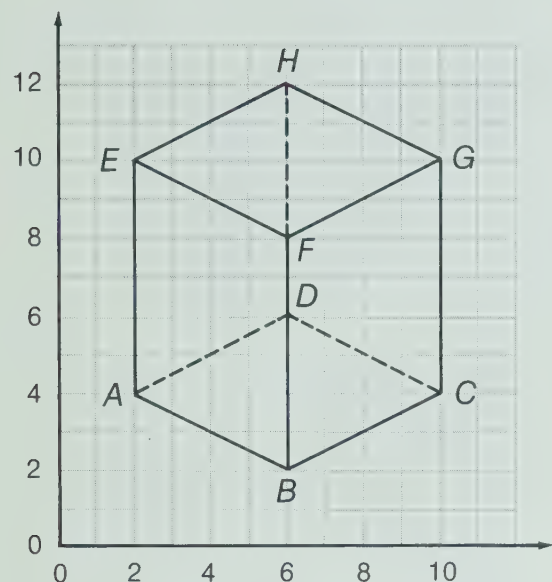


## ● Stretching and Shrinking with Coordinates

Give the coordinates for the vertices on the cube and pyramid.

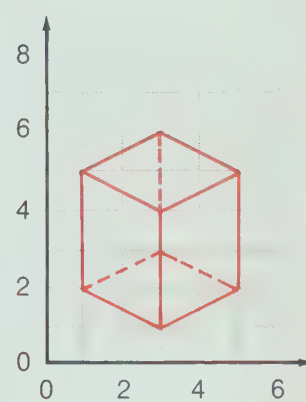
Divide all the cube coordinates by 2 to get a new set of coordinates. Double the coordinates of the pyramid.

Graph the new sets and connect the points to get a smaller cube and larger pyramid.

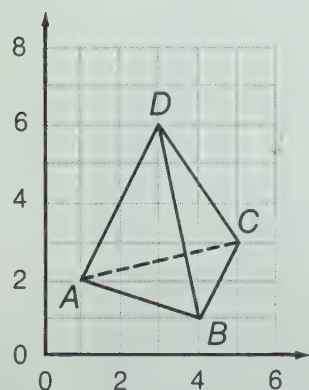


Cube coordinates		half
A: (2, 4)	→	(1, 2)
B: (6, 2)	→	(3, 1)
C: (10, 4)	→	(5, 2)
D: (6, 6)	→	(3, 3)
E: (2, 10)	→	(1, 5)
F: (6, 8)	→	(3, 4)
G: (10, 10)	→	(5, 5)
H: (6, 12)	→	(3, 6)

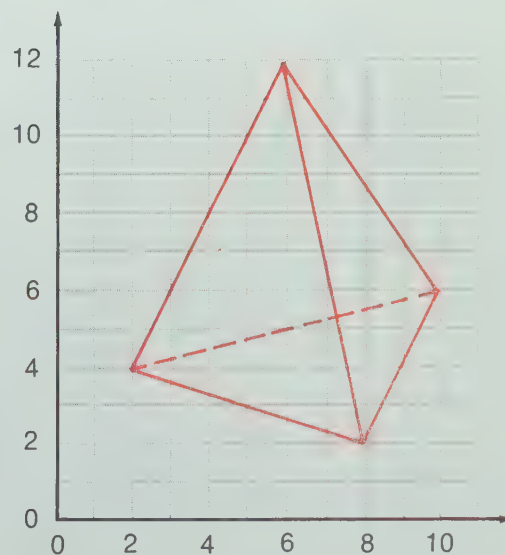
Smaller Cube



Larger Pyramid



Pyramid coordinates		double
A: (1, 2)	→	(2, 4)
B: (4, 1)	→	(8, 2)
C: (5, 3)	→	(10, 6)
D: (3, 6)	→	(6, 12)

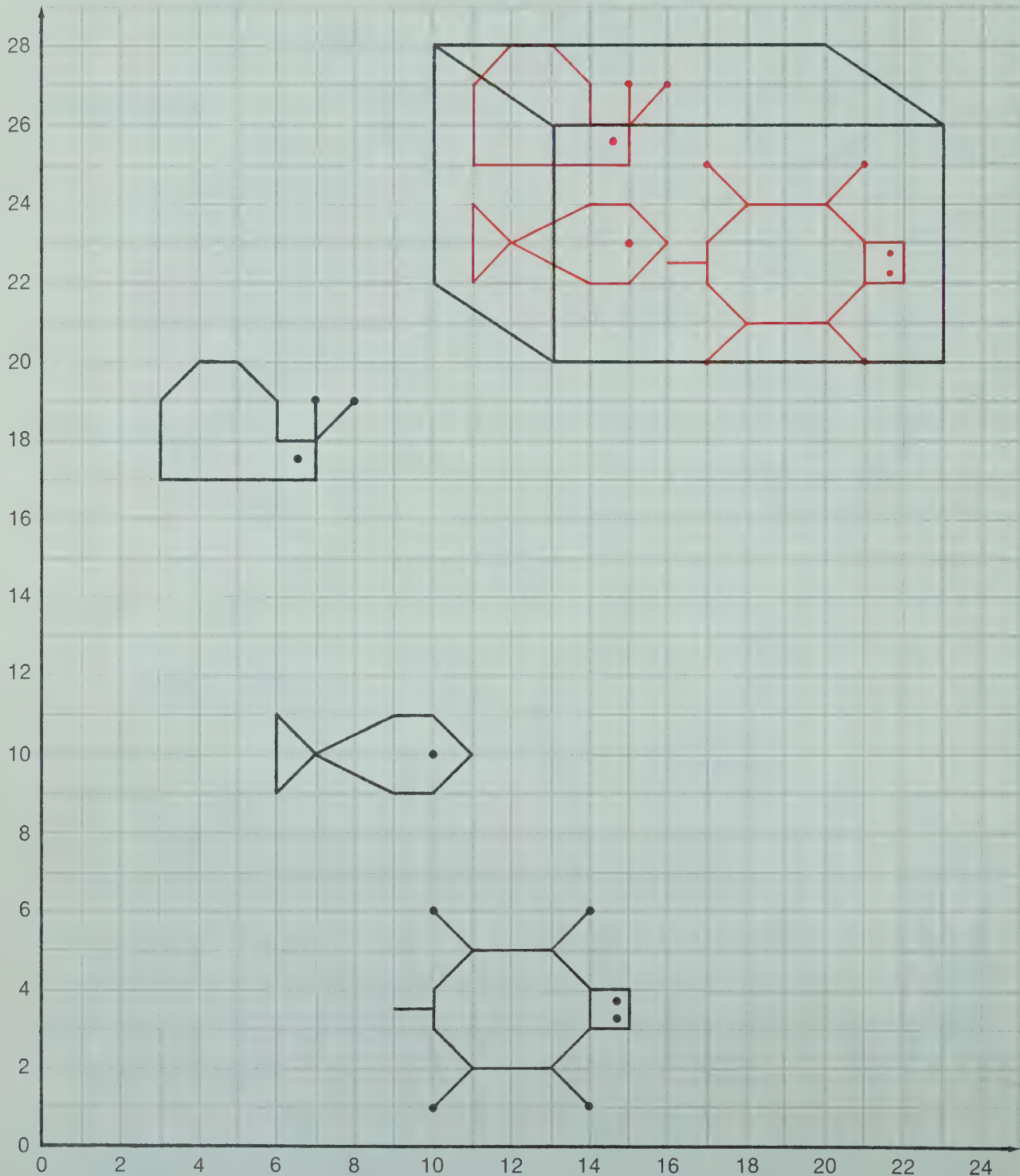


## ● Moving Figures

First, tell what moves you would make to put the fish, snail, and turtle in the tank. Then put them in the tank.

Sample answers

Moves { Fish over 5, up 13  
 Snail over 8, up 8  
 Turtle over 7, up 19

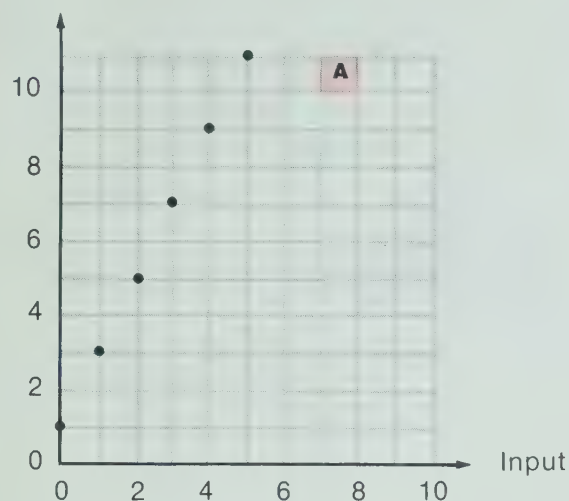


Putting the fish, snail, and turtle in the tank involves moving them so that they fall within the overall region of the tank. Answers will vary somewhat within the region and the figures may overlap each other. The moves should be given as "over and up."

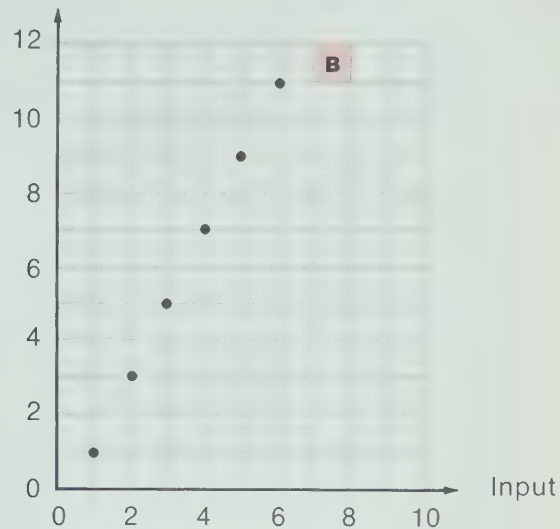
## Finding Function Rules

Complete an input-output card for each point on the graph. Then give a function rule for the set of cards.

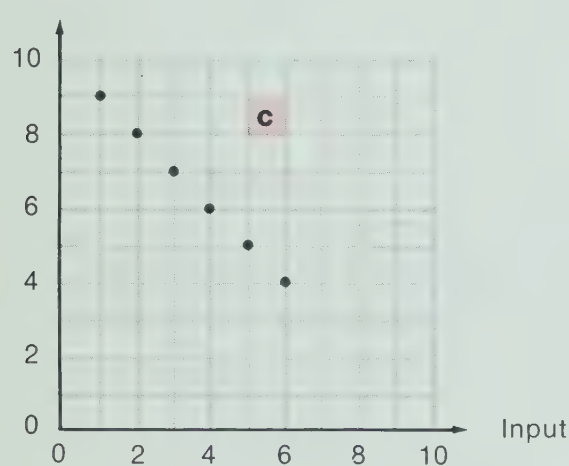
Output



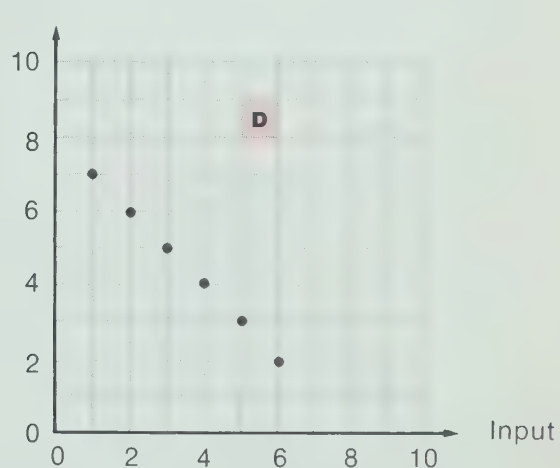
Output



Output



Output



Cards for **A**

Function Rule

**Double and add 1**

Input	Output
( 0 , 1 )	
( 1 , 3 )	
( 2 , 5 )	
( 3 , 7 )	
( 4 , 9 )	
( 5 , 11 )	

Cards for **B**

Function Rule

**Double and Subtract 1**

Input	Output
( 1 , 1 )	
( 2 , 3 )	
( 3 , 5 )	
( 4 , 7 )	
( 5 , 9 )	
( 6 , 11 )	

Cards for **C**

Function Rule

**Subtract from 10**

Input	Output
( 1 , 9 )	
( 2 , 8 )	
( 3 , 7 )	
( 4 , 6 )	
( 5 , 5 )	
( 6 , 4 )	

Cards for **D**

Function Rule

**Subtract from 8**

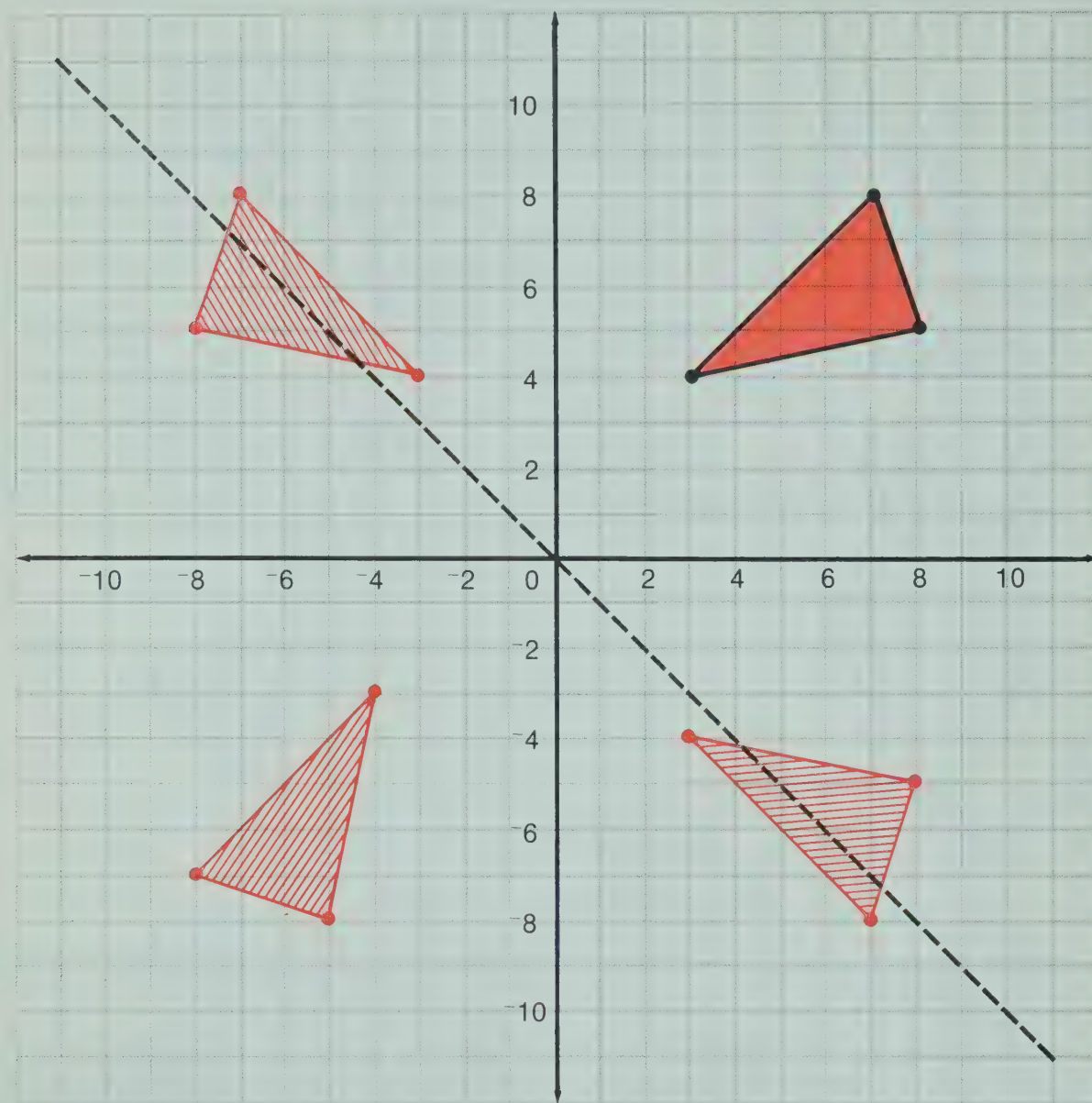
Input	Output
( 1 , 7 )	
( 2 , 6 )	
( 3 , 5 )	
( 4 , 4 )	
( 5 , 3 )	
( 6 , 2 )	

It is most important that the children give the input-output pairs prior to trying to discover the function rule.



## ● Positive and Negative Coordinates

Imagine the red triangle is covered with wet paint. If you folded the paper it would leave a mark on the other side of the fold.



1. Show the mark the red triangle will leave when you fold the paper along the given lines. Give the coordinates of the vertices of each triangle.

**A** The horizontal number line.  $(3, -4)(7, -8)(8, -5)$

**B** The vertical number line.  $(-3, 4)(-7, 8)(-8, 5)$

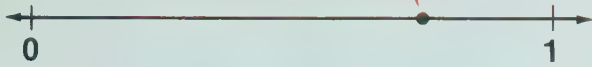
**C** The dashed line.  $(-4, -3)(-8, -7)(5, -8)$

The children can actually experience the concepts developed in this lesson by actually folding along the given lines. Then using a compass point to punch holes through the corners of the triangles, they can find where each point will fall on the other side of the fold.

Give a set of 4 equivalent fractions for the number indicated by the arrow. You may find it helpful to cut out the two rulers at the bottom of the page.

1.

$$\left\{ \frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16} \right\}$$

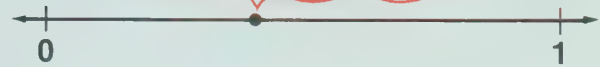


2.

$$\left\{ \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8} \right\}$$

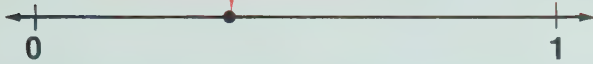


$$\left\{ \frac{2}{5}, \frac{4}{10}, \frac{6}{15}, \frac{8}{20} \right\}$$



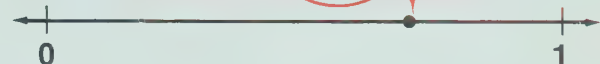
3.

$$\left\{ \frac{3}{8}, \frac{6}{16}, \frac{9}{24}, \frac{12}{32} \right\}$$



4.

$$\left\{ \frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40} \right\}$$



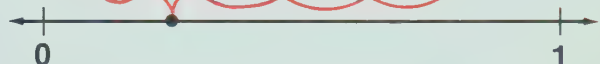
5.

$$\left\{ \frac{7}{16}, \frac{14}{32}, \frac{21}{48}, \frac{28}{64} \right\}$$



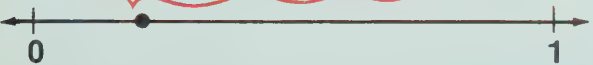
6.

$$\left\{ \frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16} \right\}$$



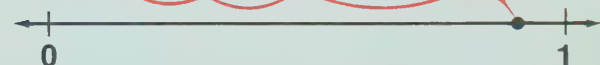
7.

$$\left\{ \frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \frac{4}{20} \right\}$$



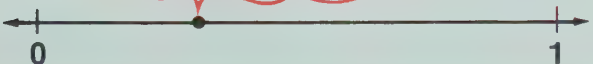
8.

$$\left\{ \frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \frac{36}{40} \right\}$$



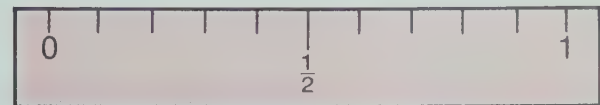
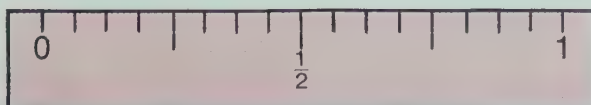
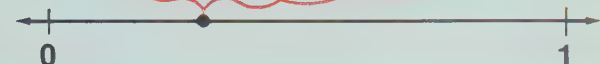
9.

$$\left\{ \frac{5}{16}, \frac{10}{32}, \frac{15}{48}, \frac{20}{64} \right\}$$



10.

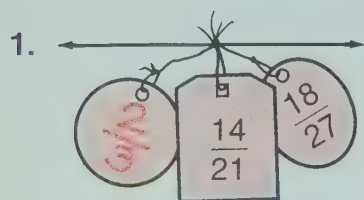
$$\left\{ \frac{3}{10}, \frac{6}{20}, \frac{9}{30}, \frac{12}{40} \right\}$$



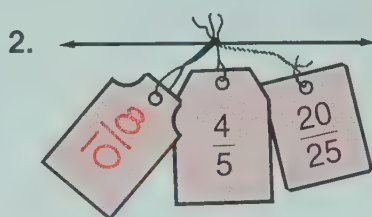
Notice that one ruler is divided into 16ths and the other is divided into tenths. Allow the children an opportunity to discover which ruler to use for any given number line.

## Fractional Number Names

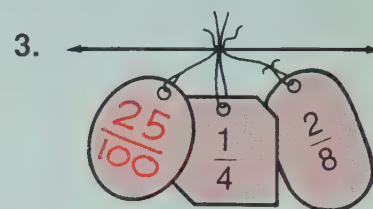
Complete each blank name tag according to directions.



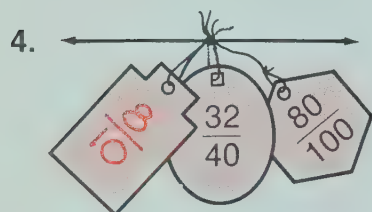
lowest terms



tenths



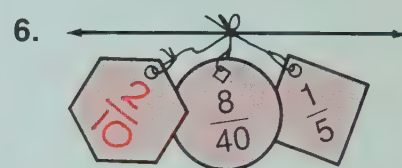
hundredths



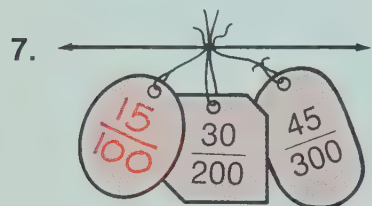
tenths



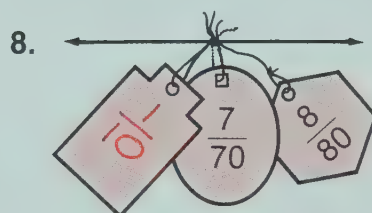
lowest terms



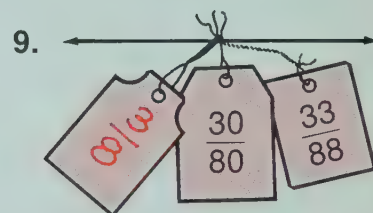
tenths



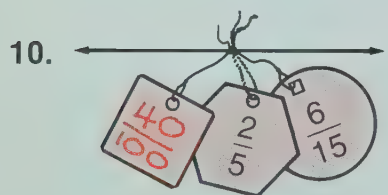
hundredths



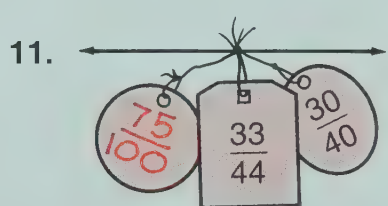
tenths



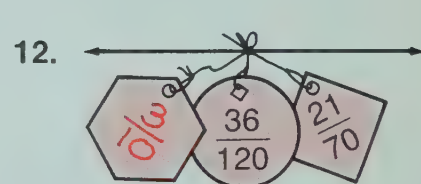
lowest terms



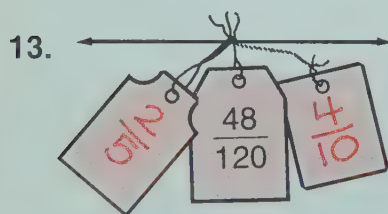
hundredths



hundredths

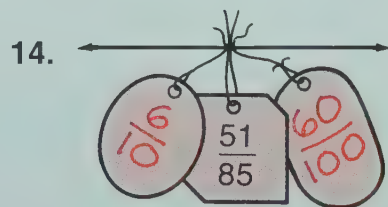


tenths



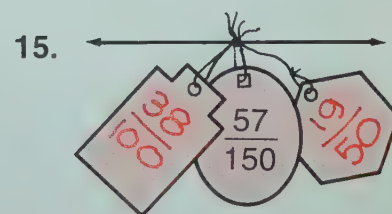
lowest terms

tenths



tenths

hundredths



hundredths

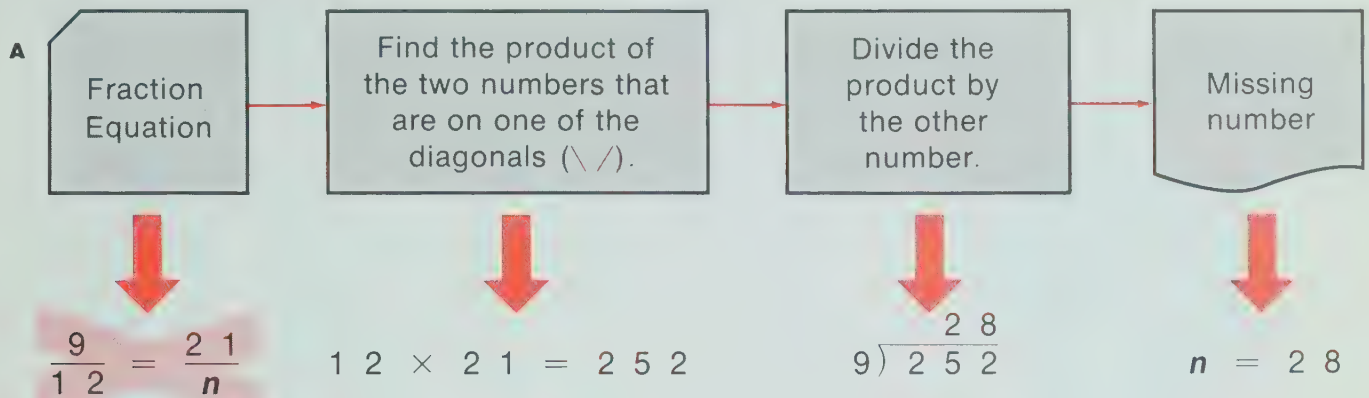
lowest terms

In most cases, it would be helpful if children found the lowest-terms fraction.



## ● Solving Equivalent Fraction Equations

1. Study the flow chart for solving equivalent fraction equations.



**B** Now you finish this one.

$$\frac{6}{9} = \frac{n}{4\ 5} \quad 6 \times 4\ 5 = 2\ 7\ 0 \quad 9 \overline{) 2\ 7\ 0} \quad n = 30$$

**C** Now try this one on your own.

$$\frac{9}{2\ 5} = \frac{2\ 4}{n} \quad 1\ 5 \times 2\ 4 = 360 \quad 9 \overline{) 360} \quad n = 40$$

2. Solve the equivalent fraction equations. The answers are somewhere in the answer box.

**A**  $\frac{6}{1\ 6} = \frac{1\ 2}{n}$  32

**B**  $\frac{1\ 4}{2\ 1} = \frac{n}{9}$  6

**C**  $\frac{n}{2\ 5} = \frac{8}{1\ 0}$  20

**D**  $\frac{5}{n} = \frac{4\ 5}{5\ 4}$  6

**E**  $\frac{6}{2\ 0} = \frac{n}{9\ 0}$  27

**F**  $\frac{5}{1\ 2} = \frac{3\ 5}{n}$  84

**G**  $\frac{6}{2\ 4} = \frac{1\ 1}{n}$  44

**H**  $\frac{1\ 4}{1\ 6} = \frac{2\ 8}{n}$  32

**I**  $\frac{n}{3\ 0} = \frac{1\ 2}{3\ 6}$  10

**J**  $\frac{2\ 1}{n} = \frac{2\ 7}{3\ 6}$  28

ANSWER BOX

32	44	20
	27	10
84	28	6

ANSWER BOX

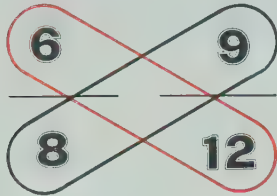
You may need to work through the top half of the page with the children to see that they understand what is required.

## Which Number is Greater?

Study the three examples for 2 fractional numbers.

### EXAMPLE 1

When the cross products are the same,



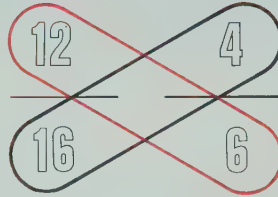
$$72 = 72$$

the fractions are equivalent and we write

$$\frac{6}{8} = \frac{9}{12}$$

### EXAMPLE 2

When the "red product" is greater,



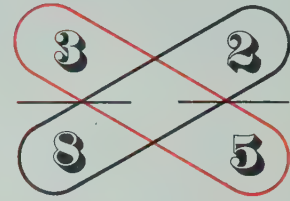
$$72 > 64$$

the first number is greater and we write

$$\frac{12}{16} > \frac{4}{6}$$

### EXAMPLE 3

When the "black product" is greater,



$$15 < 16$$

the second number is greater and we write

$$\frac{3}{8} < \frac{2}{5}$$

1. Put =, >, or < in each

A  $\frac{1}{2} > \frac{3}{8}$

B  $\frac{1}{2} < \frac{4}{7}$

C  $\frac{2}{3} < \frac{3}{4}$

D  $\frac{5}{6} > \frac{3}{4}$

E  $\frac{5}{6} > \frac{5}{8}$

F  $\frac{7}{8} > \frac{8}{10}$

G  $\frac{10}{12} = \frac{15}{18}$

H  $\frac{6}{7} < \frac{9}{10}$

I  $\frac{5}{8} < \frac{3}{4}$

J  $\frac{1}{8} < \frac{1}{7}$

K  $\frac{2}{8} > \frac{1}{7}$

L  $\frac{2}{8} < \frac{2}{7}$

M  $\frac{1}{4} < \frac{1}{3}$

N  $\frac{2}{4} > \frac{1}{3}$

O  $\frac{2}{4} < \frac{2}{3}$

Answers are given below in terms of which number (if either) is greater.

A first

B second

C second

D first

E first

F first

G equivalent

H second

I second

J second

K first

L second

M second

N first

O second

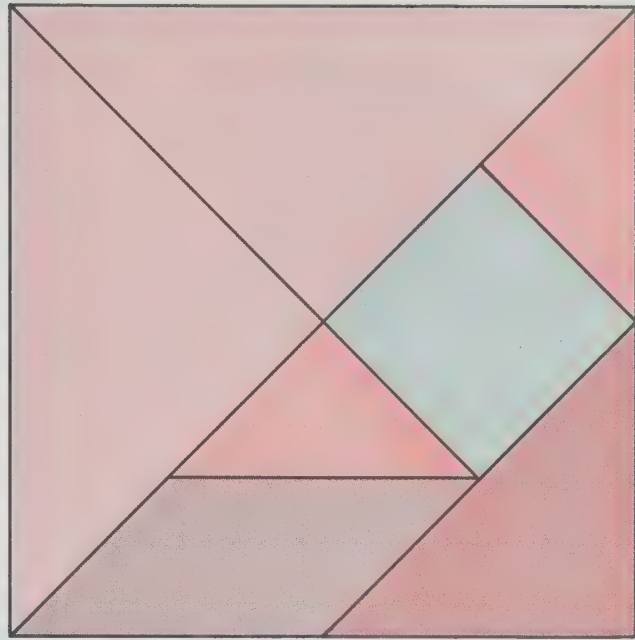
2. Can you arrange these numbers in order from smallest to largest?

$$\frac{1}{2}, \frac{1}{4}, \frac{2}{5}, \frac{3}{8}, \frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{3}, \frac{4}{7}$$

$$\frac{1}{4}, \frac{1}{3}, \frac{3}{8}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}, \frac{2}{3}, \frac{5}{6}, \frac{7}{8}$$

## ● Fractional Numbers and Area

Trace this figure and cut out the seven **tangram** pieces.



Use the small squares on the grid as units. Place your tangram pieces on the grid to help you answer these questions.

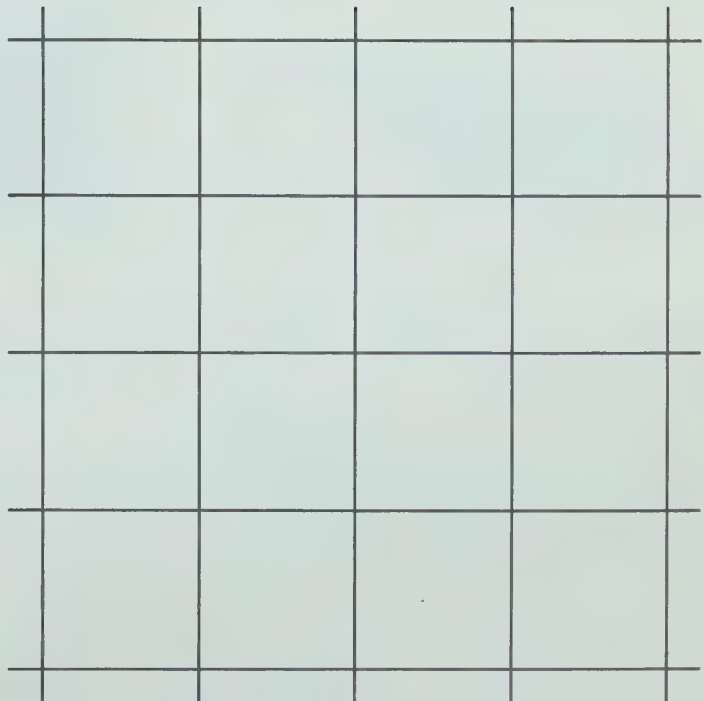
1. The area of each large triangle is 4.

2. The area of each small triangle is 1.

3. The area of the other triangle is 2.

4. The area of the square is 2.

5. The area of the parallelogram is 2.



If your answers are correct, the sum of the numbers in the 5 questions should be 11. Save your tangram pieces for page 94.

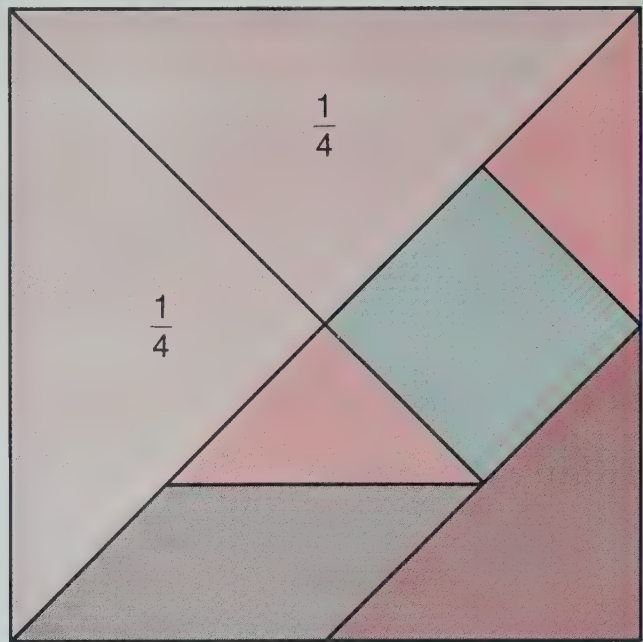
The key to finding these areas is to count "half squares." For example, each small triangle is 2 half squares and thus has an area of 1 unit.



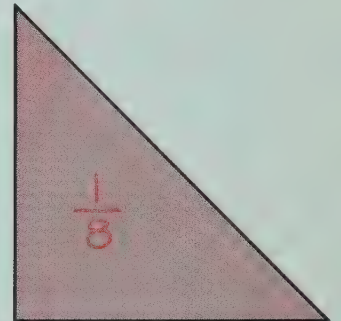
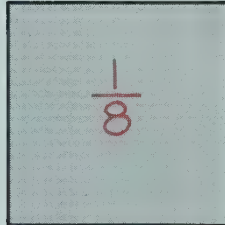
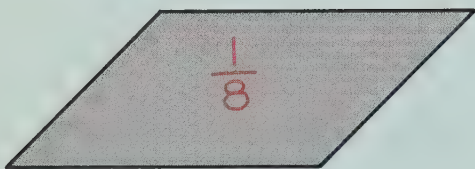
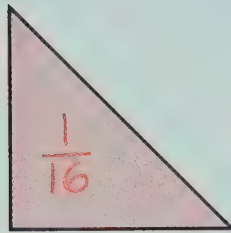
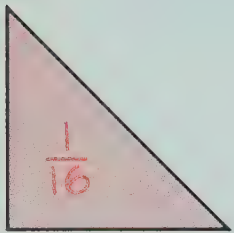
## ● Adding Fractional Numbers

Suppose the area of the large tangram square is 1. Do you see that the area of each large triangle is  $\frac{1}{4}$ ?

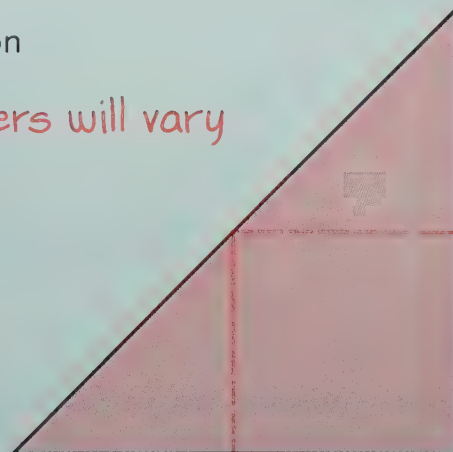
Can you find the area of each of the other tangram pieces? Write your answer on the tangram pieces pictured below. Your tangram pieces from the last lesson should help you.



Area 1



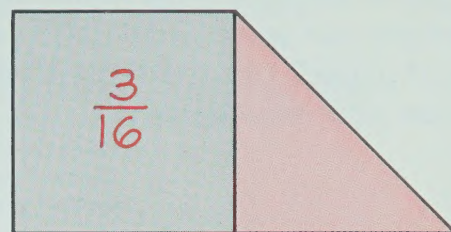
- Can you place your two small tangram pieces so they just "fit" on top of the  
**A** parallelogram?      **B** square?      **C** middle-sized triangle?
- Can you find 3 small pieces that will just fit on one of the large triangles?  
 Show how you did it on this triangle. *Answers will vary*



## ● More About Addition

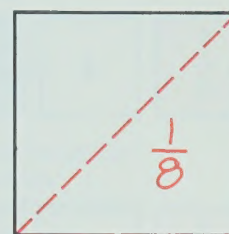
Write the fractional area that you found in the last lesson on each of your tangram pieces. Find the area of each figure by placing tangram pieces over them. Write an equation to show how you did it. Write the area inside the figure.

### EXAMPLE



$$\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$$

2.



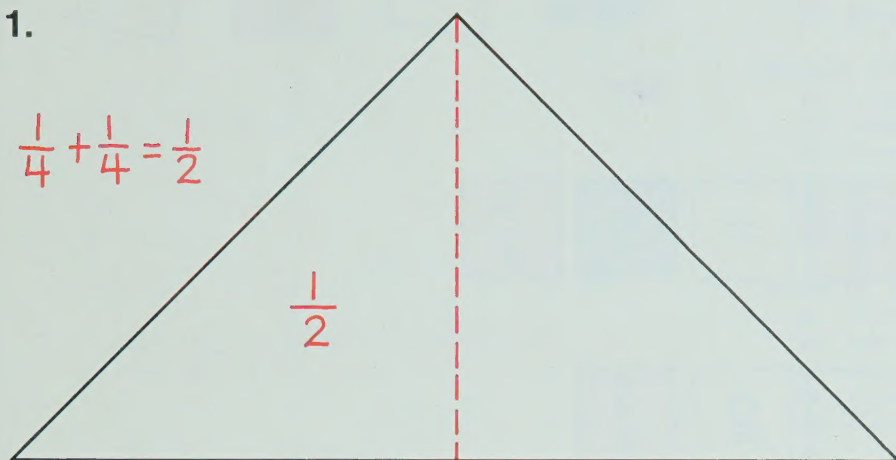
$$\frac{1}{16} + \frac{1}{16} = \frac{2}{16}$$

or

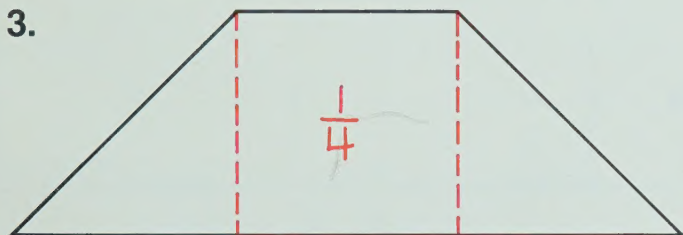
$$\frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

1.

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

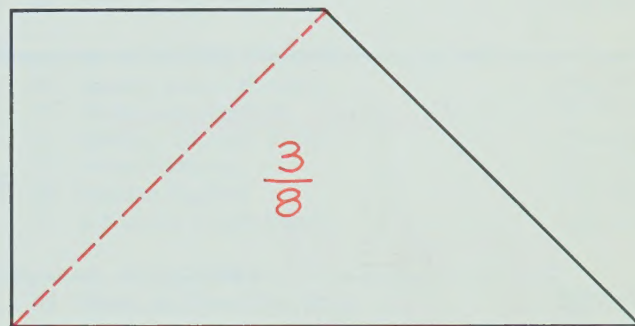


3.



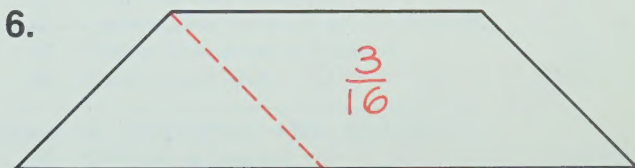
$$\frac{1}{16} + \frac{1}{8} + \frac{1}{16} = \frac{2}{8} \text{ or } \frac{1}{4}$$

4.



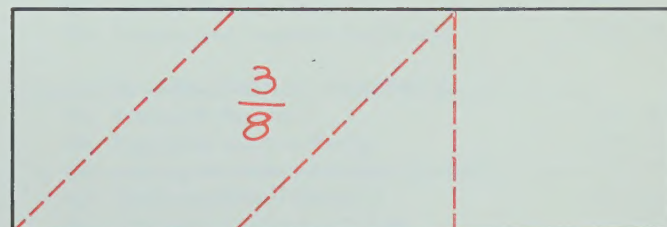
$$\frac{1}{8} + \frac{1}{4} = \frac{3}{8}$$

6.



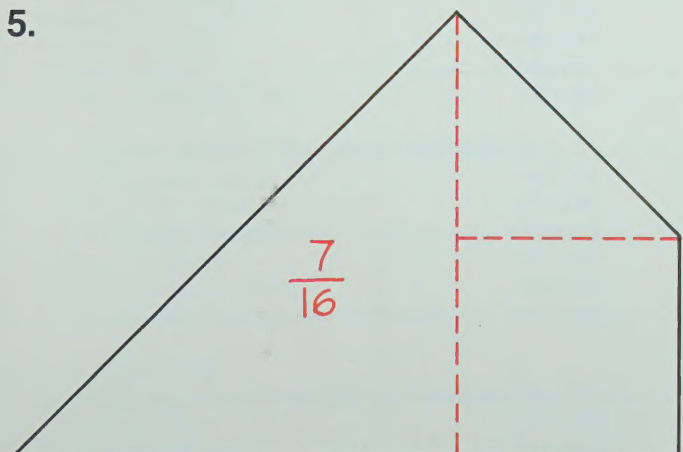
$$\frac{1}{16} + \frac{1}{8} = \frac{3}{16}$$

7.



$$\frac{1}{16} + \frac{1}{8} + \frac{1}{16} + \frac{1}{8} = \frac{3}{8}$$

5.



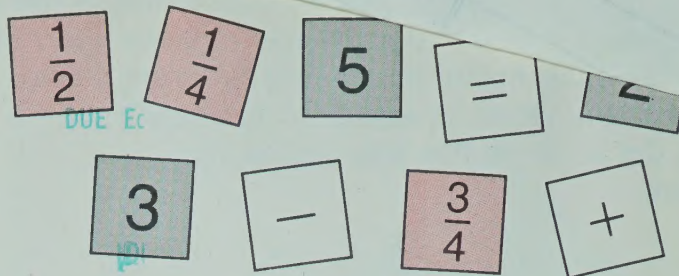
$$\frac{1}{4} + \frac{1}{16} + \frac{1}{8} = \frac{7}{16}$$

Don't try to teach a method for addition here. Rather, encourage children to "see" that  $\frac{1}{8} + \frac{1}{16}$  is the same as  $\frac{2}{16} + \frac{1}{16}$  or  $\frac{1}{16} + \frac{1}{16} + \frac{1}{16}$ .



## ● Writing Fractional Number Equations

Make slips of paper with these numerals and signs on them. How many different equations can you "write" with your slips of paper? Use the record below to show your equations.



EXAMPLES:

$$2 \frac{1}{4} + 3 \frac{1}{2} = 5 \frac{3}{4}$$

$$3 \frac{1}{4} - \frac{1}{2} = 2 \frac{3}{4}$$

Sample answers:

Record

$$2 \frac{1}{2} + 3 \frac{1}{4} = 5 \frac{3}{4}$$

$$5 \frac{3}{4} - 3 \frac{1}{4} = 2 \frac{1}{2}$$

$$5 \frac{3}{4} - 2 \frac{1}{2} = 3 \frac{1}{4}$$

$$2 \frac{3}{4} + \frac{1}{2} = 3 \frac{1}{4}$$

$$3 \frac{1}{4} - 2 \frac{3}{4} = \frac{1}{2}$$

$$3 - 2 = \frac{1}{4} + \frac{3}{4}$$

Don't try to teach a method for addition here. Rather, help children "see" that  $\frac{1}{2} + \frac{1}{4}$  is

$$\frac{2}{4} + \frac{1}{4} \text{ or } \frac{3}{4}$$





\*000028386670\*

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